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A REVISED LOWER CONFIDENCE LIMIT PROCEDURE FOR THE RELIABILITY OF COMPLEX QUASI-COHERENT SYSTEMS

by

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Submitted in partial fulfillment of the
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ABSTRACT

This thesis describes a procedure for computing a lower confidence limit on the reliability of a quasi-coherent complex system using test data on its components. The failure times of the components are assumed to have either exponential or Weibull distributions with unknown parameters. The accuracy of this procedure is evaluated using computer simulation for various system structures and sets of parameter values for the assumed distributions. This thesis is an extension of a thesis by Kah Chee Yee in that it uses a different equation for the estimate of the shape parameter in the Weibull distribution than Yee used, and it evaluates the procedure for a larger collection of system structures.

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I. INTRODUCTION

This thesis modifies an existing approximate lower confidence limit procedure for the reliability of complex systems developed by Yee [Ref. 7]. The purpose of the modification is to make the procedure more accurate for more configurations of quasi-coherent complex systems and easier to use computationally. A system is defined to be *quasi-coherent* if an increase in reliability of any one of its components does not cause a decrease in the system reliability. The components of a quasi-coherent system do not need to be statistically independent. However, throughout this thesis it is assumed that all components are statistically independent and the probability distributions of their failure times are either *exponential* or *Weibull* .

The approximate system reliability lower confidence limit procedure developed by Yee is quite accurate for series systems. In this thesis the procedure for estimating the shape parameter, β, differs from the maximum likelihood method used by Yee. The procedure developed by Varadan [Ref. 6] is used to estimate the shape parameter, β, in the Weibull distribution in this thesis.

In addition to modifying the lower confidence interval estimation procedure developed by Yee, more complex structures are examined here than in Yee's thesis. The computer programs developed by Yee were modified to examine these new structures. Also, a computer program was developed that can be used to compute the lower confidence limit for the reliability of a complex system using these procedures. The system is de-

scribed by the user in response to queries by the program. Also, the test data set is entered by the user in response to queries.

II. ESTIMATES FOR THE SHAPE PARAMETER OF A WEIBULL DISTRIBUTION

If the time to failure, X, of a device has a Weibull distribution with scale and shape parameters θ and β respectively, then its probability density function is given by

$$g(x;\theta,\beta) = (1/\theta)^{\beta}\beta x^{\beta-1}\exp\{-(x/\theta)^{\beta}\}\ , \qquad x > 0 \tag{2.1}$$

This property is stated more briefly by the phrase X is WEI(θ, β). The cumulative distribution function, CDF for X is

$$G(x;\theta,\beta) = 1 - \exp\{-(x/\theta)^{\beta}\}\ , \qquad x > 0 \tag{2.2}$$

and the survival function is

$$\overline{G}(x;\theta,\beta) = 1 - G(x;\theta,\beta) \tag{2.3}$$

It is well known, see [Ref. 6], that the random variable $Y \equiv \ln X$ has the extreme value distribution with density function f and CDF F given by

$$f(y;\mu,\sigma) = \frac{1}{\sigma}e^{(y-\mu)/\sigma}\exp\{-e^{(y-\mu)/\sigma}\}, \qquad \begin{array}{r} -\infty < y < \infty \\ -\infty < \mu < \infty \\ \sigma > 0 \end{array} \tag{2.4}$$

$$F(y;\mu,\sigma) = 1 - \exp\{-e^{(y-\mu)/\sigma}\} \tag{2.5}$$

where $\sigma = 1/\beta$, $\mu = \ln\ \theta$ and ln is the natural logarithm. In this case we write Y is EV(σ, μ).

Suppose $X_1, X_2, \ldots, X_n$ are independent random variables with a common WEI(θ, β) distribution and let $X_{(1)} < X_{(2)} < \ldots < X_{(r)}$ be the first r ordered set of the original set of n variables. Then the following equations are solved to obtain the maximum likelihood estimates (MLE) $\hat{\theta}$ and $\hat{\beta}$ for θ and β, see [Ref. 1] :

$$\frac{\sum_{j=1}^{r} X_{(j)}^{\beta} \ln X_{(j)} + (n-r) X_{(r)}^{\beta} \ln X_{(r)}}{\sum_{j=1}^{r} X_{(j)}^{\beta} + (n-r) X_{(r)}^{\beta}} - \frac{1}{\beta} = \frac{1}{r} \sum_{j=1}^{r} \ln X_{(j)} \tag{2.6}$$

$$(1/\theta)^{\beta} = \frac{r}{\sum_{j=1}^{r} X_{(j)}^{\beta} + (n-r) X_{(r)}^{\beta}} \tag{2.7}$$

Closed form expressions for $\hat{\beta}$ do not exist, and iterative procedures are used to solve for $\hat{\beta}$ and $\hat{\theta}$. Computer programs are readily available to compute $\hat{\beta}$ and $\hat{\theta}$. They require an original estimate β_0 to start an iterative process, and if it is not chosen carefully the iteration will not yield an accurate estimate for β. The MLE estimate, $\hat{\beta}$, is biased. Bain and Lee [Ref. 5] have an excellent treatment of the Weibull distribution and properties of the MLE estimates $\hat{\theta}$ and $\hat{\beta}$. Their discussion includes a table of factors (page 200) which can be used to construct nearly unbiased estimates for β .

Balakrishnan and Varadan [Ref. 6] have derived a method for estimating β and θ which does not require computer iterations. Their method will be used in this thesis to modify Yee's existing procedure that derives lower confidence limits for the reliability of complex systems that have some components whose failure times have a Weibull distribution. A summary of their results follows.

Let $Y_{(r+1)}, Y_{(r+2)}, \ldots, Y_{(n-s)}$ be a sequence of the order statistics for a random sample of size n variables each with density function given by (2.4). Then

$$Y_{(r+1)} \leq Y_{(r+2)} \leq \ldots \leq Y_{(n-s)} \tag{2.8}$$

If we let $X_{(i)} \equiv (Y_{(i)} - \mu)/\sigma$, then the likelihood function based on the censored sample is :

$$L = \frac{n!}{r!s!} \sigma^{-A} \left[F(X_{(r+1)})\right]^{r} \left[\bar{F}(X_{(n-s)})\right]^{s} \prod_{i=r+1}^{n-s} f(X_{(i)}). \tag{2.9}$$

where,

$$\begin{aligned} A &\equiv n - r - s \\ F(x) &\equiv 1 - \exp(-e^{x}) \\ \bar{F}(x) &\equiv 1 - F(x) \\ f(x) &\equiv e^{x} \exp(-e^{x}) \end{aligned}$$

The likelihood equations for μ and σ are :

$$\frac{\delta \ln L}{\delta \mu} = -\frac{1}{\sigma}\left[r \frac{f(X_{(r+1)})}{F(X_{(r+1)})} - s \frac{f(X_{(n-s)})}{\bar{F}(X_{(n-s)})} + \sum_{i=r+1}^{n-s} \frac{f'(X_{(i)})}{f(X_{(i)})}\right] = 0 \tag{2.10}$$

$$\frac{\delta \ln L}{\delta \sigma} = -\frac{1}{\sigma}\left[A + rX_{(r+1)} \frac{f(X_{(r+1)})}{F(X_{(r+1)})} - \right. \tag{2.11}$$

$$\left. sX_{(n-s)} \frac{f(X_{(n-s)})}{\bar{F}(X_{(n-s)})} + \sum_{i=r+1}^{n-s} X_{(i)} \frac{f'(X_{(i)})}{f(X_{(i)})}\right] = 0.$$

Equations (2.10) and (2.11) do not admit explicit solutions. Let, $L^{\times}$ denote the approximated likelihood function. Using a Taylor series expansion for ln L to obtain ln $L^{\times}$, the two equations can be approximated by :

$$\frac{\delta \ln L}{\delta \mu} \approx \frac{\delta \ln L^{\times}}{\delta \mu} = -\frac{1}{\sigma}[r(\gamma - \delta X_{(r+1)}) - \tag{2.12}$$

$$s(1 - \alpha_{n-s} + \beta_{n-s}X_{(n-s)}) + \sum_{i=r+1}^{n-s} (\alpha_i - \beta_i X_{(i)})] = 0$$

$$\frac{\delta \ln L}{\delta \sigma} \approx \frac{\delta \ln L^{\times}}{\delta \sigma} = -\frac{1}{\sigma}[A + rX_{(r+1)}(\gamma - \delta X_{(r+1)}) - \tag{2.13}$$

$$sX_{(n-s)}(1 - \alpha_{n-s} + \beta_{n-s}X_{(n-s)}) + \sum_{i=r+1}^{n-s} X_{(i)}(\alpha_i - \beta_i X_{(i)})] = 0.$$

See Balakrishnan and Varadan [Ref. 6 p-147]

Solving (2.12) and (2.13) for μ and σ gives their approximate maximum likelihood estimates as follows :

$$\hat{\mu} = B - \hat{\sigma} C \tag{2.14}$$

$$\hat{\sigma} = \{-D + (D^2 + 4AE)^{1/2}\}/2A \tag{2.15}$$

where ,

$$p_i \equiv \frac{i}{(n+1)} \tag{2.16}$$

$$q_i \equiv 1 - p_i \tag{2.17}$$

$$\alpha_i \equiv 1 + \ln q_i\{1 - \ln(-\ln q_i)\} \tag{2.18}$$

$$\beta_i \equiv -\ln q_i \tag{2.19}$$

$$\delta \equiv \frac{q_{r+1}}{p_{r+1}} \ln q_{r+1}\{1 + \frac{1}{p_{r+1}} \ln q_{r+1}\} \tag{2.20}$$

$$\gamma \equiv -\frac{q_{r+1}}{p_{r+1}} \ln q_{r+1}\{1 - \ln(-\ln q_{r+1})\} + \tag{2.21}$$

$$\frac{q_{r+1}}{\overset{2}{p_{r+1}}} (\ln q_{r+1})^2 \ln(-\ln q_{r+1})$$

$$A \equiv n - r - s \tag{2.22}$$

$$B \equiv \{r\delta Y_{(r+1)} + s\beta_{(n-s)} Y_{(n-s)} + \sum_{i=r+1}^{n-s} \beta_i Y_{(i)} \;\}/m \tag{2.23}$$

$$C \equiv \{r\gamma - s(1 - \alpha_{n-s}) + \sum_{i=r+1}^{n-s} \alpha_i \;\}/m \tag{2.24}$$

$$m \equiv r\delta + s\beta_{n-s} + \sum_{i=r+1}^{n-s} \beta_i \tag{2.25}$$

$$D \equiv r\gamma(Y_{(r+1)} - B) - s(1 - \alpha_{n-s})(Y_{(n-s)} - B) + \sum_{i=r+1}^{n-s} \alpha_i(Y_{(i)} - B) \tag{2.26}$$

$$E \equiv r\delta(Y_{(r+1)} - B)^2 + s\beta_{n-s}(Y_{(n-s)} - B)^2 + \sum_{i=r+1}^{n-s} \beta_i(Y_{(i)} - B)^2 \tag{2.27}$$

The approximate MLE estimates of μ and σ as defined in equations (2.14) and (2.15) are biased estimators. Balakrishnan and Varadan [Ref.6 p - 149] provide a table of constants that are called SBIAS(n,r,s). They are a function of the number n of test items placed on test, the

number r where the observations began and the number s of successive components observed. In this thesis the parameter r will always be zero, meaning that observations always start from the smallest failure time. Using their table an approximate unbiased estimate $\hat{\sigma}^{\times}$ for σ is (See Appendix A)

$$\hat{\sigma}^{\times} \equiv \frac{\hat{\sigma}}{SBIAS(n,0,s)} \tag{2.28}$$

The inverse of $\hat{\sigma}$ *is* $\hat{\beta}$,which will be a biased estimator for β . Bain [Ref. 5 p-220] provides a table of constants $B(n)$, which depend on the number of test items, n, such that $\hat{\beta}^{\times} \equiv \hat{\beta}\,\{B(n)\}$ is nearly unbiased for β.

III. DESCRIPTION OF THE LOWER CONFIDENCE LIMIT PROCEDURE

The lower confidence limit procedure developed in this thesis is a modification of the procedure in the thesis written by Yee , [Ref. 7] . The procedure in this thesis uses different estimators for the parameters β and θ in the Weibull distribution. The method of estimation presented by Mann and others [Ref. 1] and used by Yee employs the maximum likelihood estimates for β and θ which require computer iteration and a reasonably good guess for an initial value of β to perform the iteration. The estimators developed by Balakrishnan and Varadan [Ref. 6] is used in this thesis. They provide an alternative estimation procedure which does not require computer iteration methods to compute the estimates.

In this thesis we consider systems that are made up of k independent component subsystems. Systems undergo missions of some duration, say t. During a mission the system components are subject to periods of activity and inactivity. During active periods component i is subject to failure, with a hazard rate $h_i(t)$. During inactive periods the hazard rates are all zero. A component successfully completes a mission if its total operating time in $[0, t]$ exceeds some design parameter $t_i(t)$. A system completes its mission if sufficiently many of its subsystems do; the system reliability is, as usual, a function of the structure of the system.

A. SERIES SYSTEMS

1. Interval Estimation Procedure for Exponential Failure Times

Suppose a series system has k components whose failure times are statistically independent. Suppose the distribution of the failure time of

component i is exponential with failure rate λ_i. Then the system reliability R_s can be written as a function of λ_i and t_i, i = 1,2,...,k as follows :

$$R_s(t) = \exp(-\sum_{i=1}^{k} \lambda_i t_i) \tag{3.1}$$

where $t_i = t_i(t)$ is the time component i operates when the system operates for time t. Using the relationship $r_i = \lambda_i / \lambda_m$, for i = 1,2,...,k where λ_m is the failure rate of any one of the k components, equation (3.1) becomes

$$R_s(t) = \exp(-\lambda_m \sum_{i=1}^{k} r_i t_i) \tag{3.2}$$

If the values of the r_i are known and if $\hat{\lambda}_{m,U_{(\alpha)}}$ is an upper $100(1-\alpha)$ % confidence limit for λ_m, the corresponding approximate lower confidence limit for $R_s(t)$ would be :

$$\hat{R}_s(t)_{L(\alpha)} = \exp(-\hat{\lambda}_{m,U(\alpha)} \sum_{i=1}^{k} r_i t_i) \tag{3.3}$$

The equation for $\hat{\lambda}_{m,U_{(\alpha)}}$ depends on the plan for testing the components. The following case is considered in this thesis :

If n_i items of component type i are tested until f_i failures occur, i = 1,2,...,k and if $X_{i(1)}, X_{i(2)},...,X_{i(f_i)}$ denote these ordered f_i failure times then

$$\hat{\lambda}_{m,U(\alpha)} = \frac{\chi^2_{\alpha,2F}}{2\sum_{i=1}^{k} r_i T_i} \tag{3.4}$$

where T_i denotes the total test time accumulated on all n_i items of type i; i.e, $T_i = (n_i - f_i)X_{i(f_i)} + \sum_{j=1}^{f_i} X_{i(j)}$, $F = \sum_{i=1}^{k} f_i$ and $\chi^2_{\alpha,2F}$ is the $100(1-\alpha)th$

percentile point of a Chi-square distribution with 2F degrees of freedom. See Bain and Engelhardt [Ref. 3].

Values of the r_i are assumed to be unknown in this thesis. They will be estimated by $\hat{r}_i$, a nearly unbiased estimator for r_i, defined by

$$\hat{r}_i = \frac{\hat{\lambda}_i}{\hat{\lambda}_m} \tag{3.5}$$

where $\hat{\lambda}_i = (f_i - 1)/T_i$ which is an unbiased estimator for λ_i when $f_i > 1$ [Ref. 7 p. 39]. If $1/\hat{\lambda}_m$ were unbiased for $1/\lambda_m$ and if $\hat{\lambda}_m$ and $\hat{\lambda}_i$ are independent, then $\hat{r}_i$ would be an unbiased estimator for r_i. Replacing $\hat{\lambda}_m$ with $\hat{\lambda}_m f_m/(f_m - 1)$ in equation (3.5) will yield an unbiased estimator $\hat{r}_i$ for r_i. Multiplying by this constant $f_m/(f_m - 1)$ is nullified by a cancellation with the same constant in the final equation for the system reliability lower confidence limit, so equation (3.5) is used to estimate r_i. Using the estimator $\hat{r}_i$ for r_i, equation (3.4) becomes

$$\hat{\lambda}_{m,U(\alpha)} = \frac{\chi^2_{\alpha,2F}}{2\sum_{i=1}^{k} \hat{r}_i T_i} \tag{3.6}$$

It is important to note that the index m denotes the component for which $\hat{\lambda}_i = (f_i - 1)/T_i$ is the largest among all the components in the system. The corresponding equation for the $100(1 - \alpha)$ % lower confidence limit on the reliability of the series system becomes

$$\hat{R}_s(t)_{L(\alpha)} = \exp(-\hat{\lambda}_{m,U(\alpha)} \sum_{i=1}^{k} \hat{r}_i t_i) \tag{3.7}$$

2. Interval Estimation Procedure for Weibull Failure Times

Suppose a series system has k statistically independent components and suppose the failure times of all k components have Weibull distributions. Let the time to failure, X_i , of components of type i have density function

$$f_i(t_i) = \lambda_i^{\beta_i}\beta_i t_i^{\beta_i - 1}\exp\{-(\lambda_i t_i)^{\beta_i}\}\ , \quad t_i > 0 \tag{3.8}$$

Then the system reliability $R_s(t)$ can be written as

$$R_s(t) = \prod_{i=1}^{k} exp[-(\lambda_i t_i)^{\beta_i}] \tag{3.9}$$

$$= exp[-\sum_{i=1}^{k} \lambda_i^{\beta_i} t_i^{\beta_i}]$$

$$= exp[-\lambda_m^{\times}\sum_{i=1}^{k} r_i t_i^{\beta_i}]$$

where $\lambda_i^{\times} = \lambda_i^{\beta_i}$, $\lambda_m^{\times}$ is any one of the $\lambda_i^{\times}$, i = 1,2,...,k and $r_i = \lambda_i^{\times}/\lambda_m^{\times}$. If the β_i values are known, then $X_i^{\beta_i}$ will have an exponential distribution with constant failure rate $\lambda_i^{\beta_i}$ and the procedures described in Section 1 can be used to obtain $\hat{R}_s(t)_{L(\alpha)}$ with T_i defined by

$$T_i = (n_i - f_i)X_{i(f_i)}^{\beta_i} + \sum_{j=1}^{f_i} X_{i(j)}^{\beta_i} \tag{3.10}$$

Suppose β_i is unknown and $X_{i(1)},X_{i(2)},...,X_{i(f_i)}$ are the ordered failure times under failure truncated testing for component i in the system. The maximum likelihood estimates $\hat{\beta}_i$ for β_i and $\hat{\lambda}_i$ for λ_i are the solutions for β_i and λ_i in equations :

$$\frac{\sum_{j=1}^{f_i} X_{i(j)}^{\beta_i} \ln X_{i(j)} + (n_i - f_i) X_{i(r)}^{\beta_i} \ln X_{i(r)}}{\sum_{j=1}^{f_i} X_{i(j)}^{\beta_i} + (n_i - f_i) X_{i(r)}^{\beta_i}} - \frac{1}{\beta_i} = \frac{1}{f_i} \sum_{j=1}^{f_i} \ln X_{i(j)} \qquad (3.11)$$

$$\lambda_i^{\beta_i} = \frac{f_i}{\sum_{j=1}^{f_i} X_{i(j)}^{\beta_i} + (n_i - f_i) X_{i(r)}^{\beta_i}} \qquad (3.12)$$

where $X_{i(r)} = X_{i(f_i)}$ under failure truncation testing. See Mann and others [Ref. 1 p. 189-191] .

Equations (3.11) and (3.12), can not be solved in closed form. They may be solved using iterative computer methods, but in this thesis another method is used which does not need computer iterations to obtain $\hat{\beta}_i$ and $\hat{\lambda}_i$. This method was described in Chapter II. It is an approximation of the maximum likelihood estimate. Let,

$$T_{ij} = X_{ij}^{\hat{\beta}_i^{\times}} \qquad , i = 1,2,...,k \qquad j = 1,2,..., n_i \qquad (3.13)$$

In this thesis the distribution of T_{ij} is approximated by the exponential distribution with failure rate $\lambda_i^{\hat{\beta}_i^{\times}} = \lambda_i^{\times}$.

The procedures for obtaining the lower confidence limit for system reliability are similar to those in Section A. Define

$$\hat{r}_i = \frac{\hat{\lambda}_i^{\times}}{\hat{\lambda}_m^{\times}} \qquad (3.14)$$

where,

$$\hat{\lambda}_i^{\times} = \frac{f_i}{T_i} = \frac{f_i}{\sum_{j=1}^{n_i} T_{ij}} \quad , \ i = 1,2,\ldots,k \tag{3.15}$$

and

$$\hat{\lambda}_m^{\times} = \max_i \hat{\lambda}_i^{\times} .$$

Then equation (3.14) becomes

$$\hat{r}_i = \hat{\lambda}_i^{\times} \left(\frac{T_m}{f_m} \right) \tag{3.16}$$

The approximate $100(1-\alpha)$ % upper confidence limit for $\lambda_m^{\times}$ is :

$$\hat{\lambda}_{m,U(\alpha)}^{\times} = \frac{\chi^2_{\alpha,2F^{\times}}}{2\sum_{i=1}^{k} \hat{r}_i T_i} \tag{3.17}$$

where,

$$F^{\times} = \sum_{i=1}^{k} f_i \ . \tag{3.18}$$

The corresponding approximate $100(1-\alpha)$ % lower confidence limit for the reliability $R_s(t)$ of the series system is given by :

$$\hat{R}_s(t)_{L(\alpha)} = exp[- \hat{\lambda}_{m,U(\alpha)}^{\times} \sum_{i=1}^{k} \hat{r}_i t_i^{\hat{\beta}_i^{\times}}] \tag{3.19}$$

The accuracies of these approximate confidence interval procedures were evaluated by using computer simulations which are described in the next

chapter. During this evaluation process, the degrees of freedom in the expression $\chi^2_{\alpha,2F^\times}$ in equation (3.17), was increased and decreased from the defined values of $F^\times$ given by this equation. The purpose of these modifications was to find more accurate lower confidence limit procedures. The specific increases and decreases are described in chapter IV. The results show that for some cases the procedure with modified degrees of freedom is more accurate.

B. PARALLEL SYSTEMS

An active parallel one-out-of-k system is defined as a system consisting of k subsystems such that system failure occurs when and only when all k subsystems fail. Equivalently, the system is successful when at least one of its subsystems is successful. Such a parallel system is said to be an active redundant system of order k. It can be shown that all of the properties given in the preceding section for serial systems can be dualized to the corresponding properties for parallel systems by replacing any event by its complementary event.

Let the parallel system be made up of k independently operating components, each with an exponentially distributed failure time and failure rate λ_i ,i = 1,2,...,k. Then the system reliability, $R_s(t)$, can be written as a function of λ_i, i = 1,2,...,k as follows :

$$R_s(t) = 1 - \prod_{i=1}^{k} [\ 1 - \exp(-\lambda_i t_i)\ ] \qquad (3.20)$$

Using equations that have been derived in the preceding section; i.e., in the equations (3.5),(3.6),(3.14),(3.15),(3.16) and (3.17), the corresponding equations for the approximate $100(1-\alpha)$ % lower confidence limit for the reliability of a parallel system is

$$\hat{R}_s(t)_{L(\alpha)} = 1 - \prod_{i=1}^{k} [1 - \exp(-\hat{\lambda}_{m,U(\alpha)} \hat{r}_i t_i)] \tag{3.21}$$

for exponential failure times.

For Weibull failure times the approximate $100(1-\alpha)$ % lower confidence limit for the reliability of a parallel system is

$$\hat{R}_s(t)_{L(\alpha)} = 1 - \prod_{i=1}^{k} [1 - \exp(-\hat{\lambda}^{\times}_{m,U(\alpha)} \hat{r}_i t_i^{\hat{\beta}_i^{\times}})] \tag{3.22}$$

C. SERIES-PARALLEL SYSTEMS

Series-parallel or parallel-series systems come in many varieties. One example of an active series-parallel system is defined as a system that is comprised of k subsystems in series connected to d subsystems in parallel or vice-versa. The reliability block diagram of an example system can be seen in Figure 3.1.

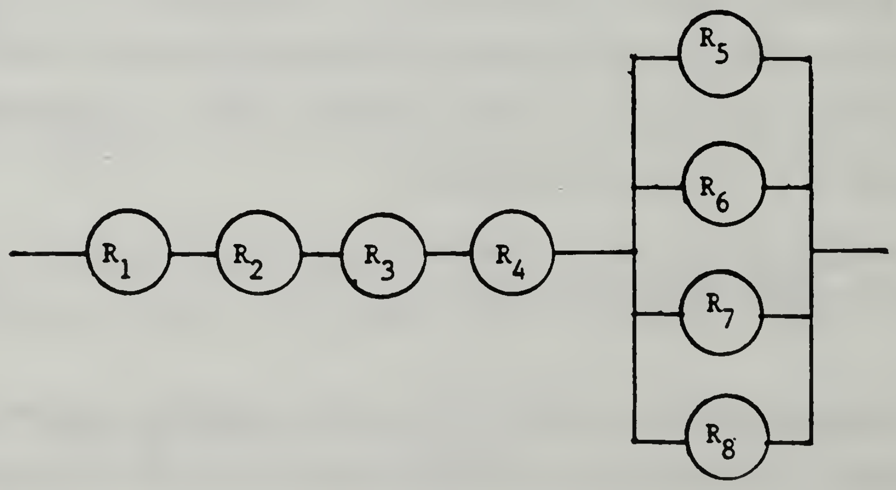

Fig.3.1 Block Diagram Of Series-parallel System

The system reliability $R_s(t)$ of this series-parallel system can be written

$$R_s(t) = \Big[\exp(-\sum_{i=1}^{k} \lambda_i t_i)\Big]\Big[1 - \prod_{i=1}^{d} [1 - \exp(-\lambda_i t_i)]\Big] \tag{3.23}$$

if the failure time of each component is exponential, and

$$R_s(t) = \Big[\exp(-\sum_{i=1}^{k} \lambda_i^{\beta_i} t_i^{\beta_i})\Big]\Big[1 - \prod_{i=1}^{d} [1 - \exp(-\lambda_i^{\beta_i} t_i^{\beta_i})]\Big] \tag{3.24}$$

if the failure time of each component has a Weibull distribution.

Using the same approach as those used for series systems and parallel systems for estimating the parameters, the corresponding equations for an approximate $100(1-\alpha)$ % lower confidence limits for the reliability of the series-parallel system are

$$\hat{R}_s(t)_{L(\alpha)} = \Big[\exp(-\hat{\lambda}_{m,U(\alpha)} \sum_{i=1}^{r} \hat{r}_i t_i)\Big]. \tag{3.25}$$

$$.[1 - \prod_{i=r+1}^{k} [1 - \exp(-\hat{\lambda}_{m,U(\alpha)} \hat{r}_i t_i)]]$$

and

$$\hat{R}_s(t)_{L(\alpha)} = \Big[\exp(-\hat{\lambda}^{\times}_{m,U(\alpha)} \sum_{i=1}^{r} \hat{r}_i t_i^{\hat{\beta}_i^{\times}})\Big]. \tag{3.26}$$

$$.[1 - \prod_{i=r+1}^{k} [1 - \exp(-\hat{\lambda}^{\times}_{m,U(\alpha)} \hat{r}_i t_i^{\hat{\beta}_i^{\times}})]]$$

IV. COMPUTER SIMULATIONS

TEST PLAN : TESTING n_i COMPONENTS UNTIL f_i FAIL (RETP)

RETP is a program written in FORTRAN, on the Amdahl mainframe computer. It performs the computer simulations that assess the accuracy of the lower confidence limit procedure for system reliability. A documentation of this program and its associated subroutines is included in Appendix C.

The program accepts input parameters via an input file INI.DAT. For each replication, it generates the failure times for all the component items included in the test plan using a uniform random number generating subroutine LRNDPC. The program determines the total test time accumulated for each component in the system and computes the estimates of the key parameters and the consequent lower confidence limit for system reliability for that replication. This is done for various system configurations; i.e., series systems (Fig. 4.1), parallel systems (Fig. 4.2), parallel-series systems (Fig. 4.3) and a more general series-parallel system (Fig. 4.4). For each specific system configuration and set of input parameters, the process is repeated 1000 times. When all replications are done, the routine EVAL processes the lower confidence limit estimates from all 1000 replications and determines the two measures of accuracy for the run, namely RSLOW and LEVEL.

RSLOW is the $100(1-\alpha)$ percentile of the *ordered* set of lower confidence limits from the 1000 replications computed in a run. The true reliability of the system is RS. The closer RSLOW is to RS, the greater the accuracy of the procedure under evaluation in the run. If the proce-

dure is exact, RSLOW will be equivalent to RS. To be conservative, RSLOW should be lower than RS.

LEVEL measures the proportion of 1000 lower confidence limits, from a run with 1000 replications, which are *lower* than the true system reliability RS. The closer LEVEL is to the specified confidence level for the procedure, $1-\alpha$, the better the procedure. Values of LEVEL greater than $(1-\alpha)$ reflect an under-estimation of RS which is conservative. Values of LEVEL less than $1-\alpha$ signal an over-estimation of RS which may be undesirable.

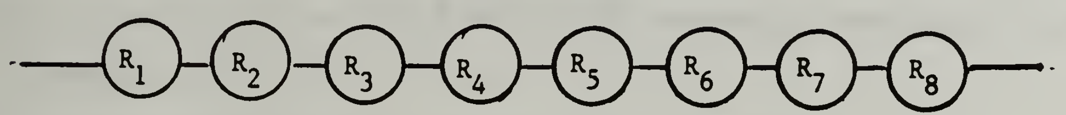

Fig. 4.1 Block Diagram Of Series System

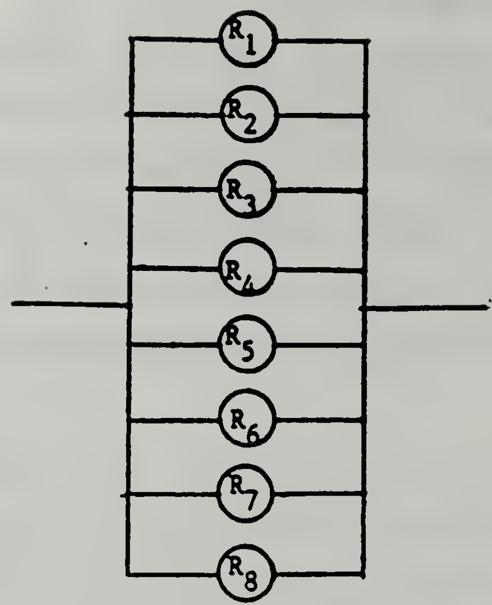

Fig. 4.2 Block Diagram Of Parallel System

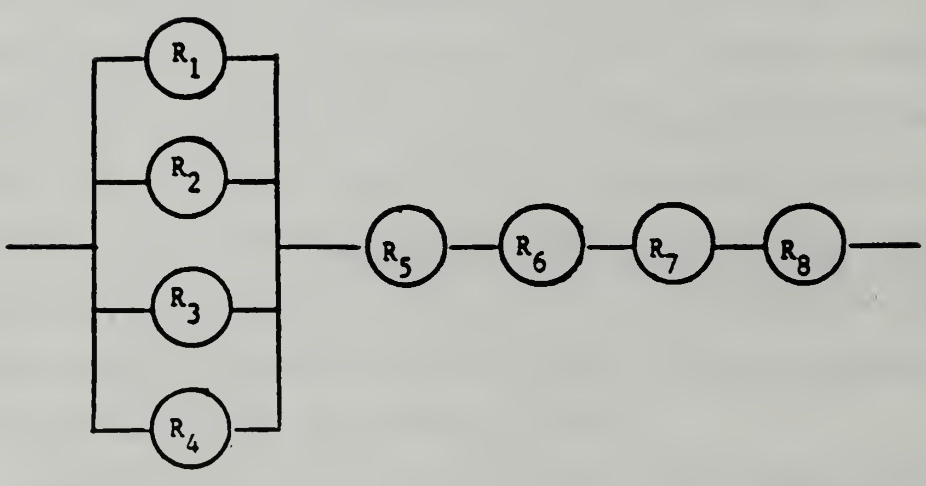

Fig. 4.3 Block Diagram Of Parallel-series System

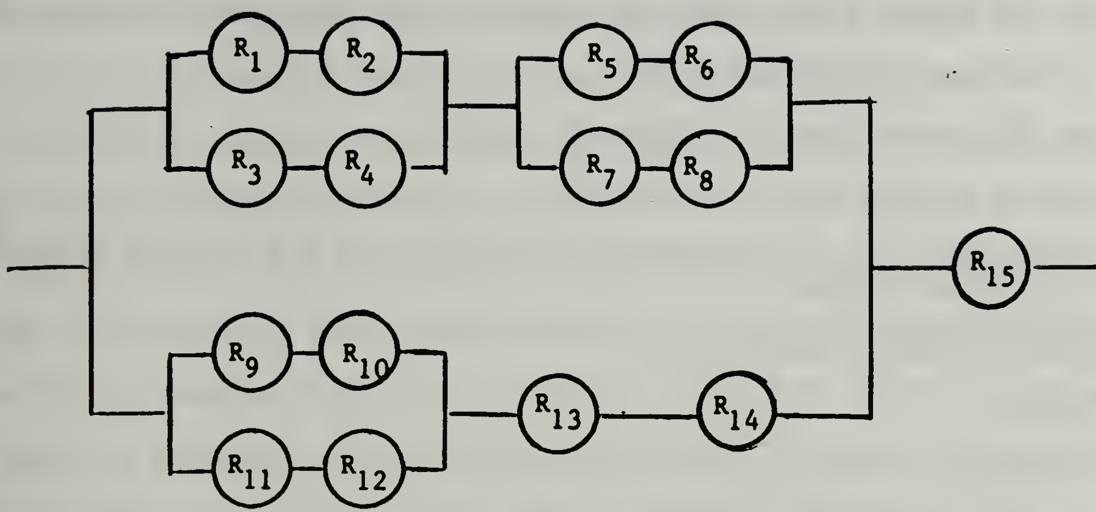

Fig. 4.4 Block Diagram Of General Series-parallel System

Simulation runs are performed using RETP for all combinations of failure time distributions and levels of key input parameters listed below.

1. System

   a. 8 Exponential (Exp) components in Series (Case 1)

   b. 8 Weibull (Wei) components in Series (Case 2)

   c. 4 Exp and 4 Wei (Mixed) components in Series (Case 3)

   Results from these simulation runs are put into tabular form. By observing the most accurate result for different degrees of freedom, an approximate equation for obtaining the Degrees of Freedom can be derived, see Appendix B. Using this equation we then performed simulation runs for other systems for RETP.

   d. 8 Exponential components in Parallel (Case 4)

   e. 8 Weibull components in Parallel (Case 5)

   f. 4 Exp and 4 Wei (Mixed) components in Parallel (Case 6)

   g. 8 Exponential components in Parallel-series (Case 7)

   h. 8 Weibull components in Parallel-series (Case 8)

i. 4 Exp and 4 Wei (Mixed) components in Series-parallel (Case 9)

j. 5 Exp and 10 Wei (Mixed) components in Series-parallel (Case 10)

2. True System Reliability (RS)

   a. Hi (greater than 0.9) (Type A)

   b. Lo (greater than 0.8) (Type B)

   See pages 9-11 for description of the values of λ_i and β_i, for each of the above 10 cases.

3. Level of Significance (α)

   a. 0.1

   b. 0.2

4. Degrees of Freedom (DF) for the x^2 statistic as a function of the total number of failed test components (NFC) and total number of system components (NCOMP), (For cases 1,2 and 3 only).

   a. DF = 2NFC

   b. DF = 2(NFC + NCOMP)

   c. DF = 2(NFC - NCOMP)

   d. DF = 2NFC - NCOMP

      For case 4 up to 10 the equation derived in Appendix B was used. This equation is

   e. DF = 2NFC + 0.5(NC/NF)

5. Test Plan for each component.

   a. Test 10 until 10 failures

   b. Test 15 until 15 failures

   c. Test 15 until 11 failures

   d. Test 15 until 7 failures

   e. Test 15 until 3 failures

For the 8 *exponential* components in series in Case 1, the mission time for each of the components was chosen to be 10 hours. The chosen values of the scale parameters, λ_i , are different depending on whether the system

is highly reliable (Type A system) or one with a lower reliability (Type B system). The ratios between the largest and the smallest failure rate were chosen to be 8 and 4.5 respectively for Type A and Type B systems.

For the 8 *Weibull* components in series in Case 2, the mission time for each of the components was chosen to be 10 hours. In order to obtain a highly reliable system (Type A) or lower reliability system (Type B) the scale parameters, λ_i, were chosen differently. The ratios between the largest and the smallest failure rate were chosen to be 8 for both system types. The shape parameter, β_i, was chosen to be 2 for all subcases. The program will accomodate any value greater than zero for the shape parameter.

In Case 3, which is a mixture of *exponential* and *Weibull* components, the mission time for each of the components was chosen to be equal to 10 hours. The scale parameter for each component type was chosen so that the ratios between the largest and the smallest failure rate were 4 for both component types. The shape parameter for each of the Weibull components is chosen to be 2 for all cases.

In Case 4 (8 *exponential* components in parallel), the mission time for each of the components was chosen to be 10 hours. The chosen values of the scale parameters, λ_i , were different depending on whether the system is highly reliable (Type A system) or one with a lower reliability (Type B system). The ratios between the largest and the smalllest failure rate were chosen to be 1.525 and 2.575 respectively for Type A and Type B systems.

For the 8 *Weibull* components in parallel in Case 5, no change was made for the mission time for each of the components as described in the previous paragraph. The ratios between the largest and the smallest failure

rate were chosen to be 1.28 for Type A systems and 1.63 for Type B systems. The shape parameter, β_i, was chosen to be 2 for all subcases.

For a mixture of *exponential* and *Weibull* components in parallel, the mission time is still the same as before which is 10 hours. The values of the scale parameters, λ_i, for the exponential components were chosen to be equal to those for the Weibull components. The ratios between the largest and the smallest failure rates were chosen to be 1.3 and 2.2 for Type A and Type B systems respectively. The shape parameter, β_i, for a Weibull component was chosen to be 2 for all subcases.

In Case 7, the 8 *exponential* components in parallel-series (Figure 4.3), the mission time for each of the components was chosen to be 10 hours. The chosen values of the scale parameters, λ_i were different depending on whether the system is highly reliable (Type A system) or one with a lower reliability (Type B system). The ratios between the largest and the smallest failure rate were chosen to be 8 for both Type A and Type B systems.

For the 8 *Weibull* components in parallel-series (Figure 4.3) in Case 8, no change was made for the mission time for each of the components which is 10 hours. The ratios between the largest and the smallest failure rate were chosen to be 8 for both systems. The shape parameter, β_i, was set at 2 for all subcases.

Case 9 is a mixture of *exponential* and *Weibull* components in parallel-series (Figure 4.3). The mission time for all components is 10 hours. The values of the scale parameters, λ_i, for exponential components were chosen to be equal to those for Weibull components. The ratios between the largest and the smallest failure rate were chosen to be

4 for both component types and both systems. The shape parameter, β_i, was set at 2 for all subcases.

Case 10 is a mixture of *exponential* and *Weibull* components in a more general series-parallel configuration (Figure 4.4). The mission time for each of the components was chosen randomly. Similarly, the scale parameter for each of the components and shape parameter for each of the Weibull components were chosen randomly. This was done for both systems.

Each simulation run of 1000 replications results in an output file OUT.DAT. The raw output from all the RETP runs are summarized in tabular form and placed in Appendix E. Each Table from Table 1A to Table 3B corresponds to a specific run case and system type combination using various degrees of freedom. Each Table from Table 4A to Table 10B corresponds to a specific run case and system type combination using only one specific degree of freedom.

V. ANALYSIS OF SIMULATION RESULTS

After simulation runs were completed for selected cases, the results were analyzed for comparisons with results that had been obtained by Yee [Ref. 7] .

For each case, simulations were run with different degrees of freedom for the chi-square percentile point. This was done to determine if a formula for the degrees of freedom could be developed that would yield a more accurate lower confidence limit procedure.

TEST PLAN : TESTING n_i COMPONENTS UNTIL f_i FAIL (RETP)

When all components of the system have exponentially distributed failure times, the lower confidence limit procedures in this thesis are nearly the same as those developed by Yee. Consequently analysis of the simulation results will be discussed primarily for cases when some components of the system have failure times that have a Weibull distribution.

A comparison of the four values of RSLOW, as in Table 1A for each of 5 sampling plans (denoted by S/N), reveals that the lower confidence limit procedure with degrees of freedom equal to 2NFC - NCOMP is the most accurate lower confidence limit procedure. In S/N 2, for example, the RSLOW value of 0.9298 (using 2NFC - NCOMP degrees of freedom) is the largest such value below the RS value of 0.9305. The value of RSLOW above RS are optimistic and not as desirable as values of RSLOW which are equi-distant below RS.

Table 2A and Yee's Table 2A provide a comparison of the simulation results for eight *Weibull* components in series using the same key parameters and the same mission times for each component as used by Yee.

When testing 15 components until all fail, Yee's procedure and the procedure in this thesis have nearly equal accuracy. But when testing 15 items until 7 fail, the procedure written in this thesis gives a more accurate value of RSLOW than the one from Yee's procedure. In this case, Yee's procedure yields values of RSLOW that are all above RS. In both Yee's procedure and the one used in this thesis, the degrees of freedom of 2(NFC - NCOMP) gave the most accurate results among the four choices of degrees of freedom. It can also be seen that for more truncated testing, such as testing 15 items until 3 fail, the value of RSLOW tends to be higher and they are all slightly above RS.

Table 3A displays the results of Case 3 for a type A system. The shape parameter, β, for the failure times with Weibull distribution was set equal to 2. The value of RSLOW resulting from the procedure developed in this thesis is more accurate than that for Yee's procedure for various degrees of freedom. Inspection of Table 3A reveals that the procedure corresponding to degrees of freedom 2(NFC + NCOMP) is reasonably accurate for all 5 simulation cases and for both 90% and 80% confidence levels.

Since the accuracy of the procedure depends on the extent of the truncation in the testing, a method was developed for choosing the degrees of freedom that includes a term with the ratio NC/NF, where NC is the number of components placed on test and NF is the number of failed components. This makes the procedure dependent on the amount of truncation in the testing. Using information from all the simulations and the degrees of freedom that gave the most accurate values of RSLOW for all cases simulated, a formula for the degrees of freedom, DF, was developed using least squares methods.

The equation is DF = 2NFC + 1.1(NC/NF) (see Appendix B for the derivation of this equation). Applying the equation to obtain RSLOW and observing the results for all cases simulated, resulted in a small modification to the above equation to yield the final equation DF = 2NFC + 0.5(NC/NF). This final equation was used for the remaining cases that were simulated in RETP when any components had failure times with Weibull distribution.

Table 1A : 8 Exp in Series, RS = 0.9305 (Hi)
min λ = 0.0002 f/hr, max λ = 0.0016 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC = 80	2NFC (160)	0.1	0.9262	0.9630
			0.2	0.9256	0.9220
		2(NFC+ NCOMP) (176)	0.1	0.9196	0.9970
			0.2	0.9188	0.9910
		2NFC- NCOMP (152)	0.1	0.9296	0.9209
			0.2	0.9291	0.8430
		2(NFC- NCOMP) (144)	0.1	0.9329	0.8400
			0.2	0.9325	0.7300
2	Test 15 until 15 failed NFC = 120	2NFC (240)	0.1	0.9277	0.9550
			0.2	0.9273	0.9080
		2(NFC+ NCOMP) (256)	0.1	0.9233	0.9899
			0.2	0.9228	0.9750
		2NFC- NCOMP (232)	0.1	0.9298	0.9159
			0.2	0.9296	0.8329
		2(NFC- NCOMP) (224)	0.1	0.9321	0.8440
			0.2	0.9318	0.7470
3	Test 15 until 11 failed NFC = 88	2NFC (176)	0.1	0.9268	0.9550
			0.2	0.9262	0.9159
		2(NFC+ NCOMP) (192)	0.1	0.9208	0.9960
			0.2	0.9200	0.9880
		2NFC- NCOMP (168)	0.1	0.9298	0.9159
			0.2	0.9293	0.8430
		2(NFC- NCOMP) (160)	0.1	0.9328	0.8430
			0.2	0.9324	0.7350

Table 1A : 8 Exp in Series, RS = 0.9305 (Hi) (Cont...)
min λ = 0.0002 f/hr, max λ = 0.0016 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
4	Test 15 until 7 failed NFC=56	2NFC (112)	0.1	0.9241	0.9700
			0.2	0.9229	0.9310
		2(NFC+ NCOMP) (128)	0.1	0.9145	0.9980
			0.2	0.9130	0.9940
		2NFC- NCOMP (104)	0.1	0.9289	0.9190
			0.2	0.9280	0.8530
		2(NFC- NCOMP) (96)	0.1	0.9338	0.8350
			0.2	0.9331	0.7200
5	Test 15 until 3 failed NFC=24	2NFC (48)	0.1	0.9145	0.9859
			0.2	0.9129	0.9750
		2(NFC+ NCOMP) (64)	0.1	0.8907	1.0000
			0.2	0.8877	1.0000
		2NFC- NCOMP (40)	0.1	0.9268	0.9439
			0.2	0.9260	0.8600
		2(NFC- NCOMP) (32)	0.1	0.9394	0.7530
			0.2	0.9394	0.6339

Yee's results
Table 1A : 8 Exp in Series, RS = 0.931 (Hi)
min λ = 0.0002 f/hr, max λ = 0.0016 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 5 until 5 failed NFC=40	2NFC (80)	0.1	0.919	0.982
			0.2	0.919	0.960
		2(NFC+ NCOMP) (96)	0.1	0.906	1.000
			0.2	0.905	0.999
		2NFC- NCOMP (72)	0.1	0.927	0.949
			0.2	0.927	0.880
		2(NFC- NCOMP) (64)	0.1	0.934	0.821
			0.2	0.934	0.702
2	Test 15 until 15 failed NFC=120	2NFC (240)	0.1	0.928	0.955
			0.2	0.927	0.908
		2(NFC+ NCOMP) (256)	0.1	0.923	0.990
			0.2	0.923	0.975
		2NFC- NCOMP (232)	0.1	0.930	0.916
			0.2	0.930	0.833
		2(NFC- NCOMP) (224)	0.1	0.932	0.844
			0.2	0.932	0.747
3	Test 15 until 11 failed NFC=88	2NFC (176)	0.1	0.927	0.955
			0.2	0.926	0.916
		2(NFC+ NCOMP) (192)	0.1	0.921	0.996
			0.2	0.920	0.988
		2NFC- NCOMP (168)	0.1	0.930	0.916
			0.2	0.929	0.843
		2(NFC- NCOMP) (160)	0.1	0.933	0.843
			0.2	0.932	0.735

Yee's results

Table 1A : 8 Exp in Series, RS = 0.931 (Hi) (Cont...)
min λ = 0.0002 f/hr, max λ = 0.0016 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
4	Test 15 until 7 failed NFC=56	2NFC (112)	0.1	0.924	0.970
			0.2	0.923	0.931
		2(NFC+ NCOMP) (128)	0.1	0.915	0.998
			0.2	0.913	0.994
		2NFC- NCOMP (104)	0.1	0.929	0.919
			0.2	0.928	0.853
		2(NFC- NCOMP) (96)	0.1	0.934	0.835
			0.2	0.933	0.720
5	Test 15 until 3 failed NFC=24	2NFC (48)	0.1	0.915	0.986
			0.2	0.912	0.975
		2(NFC+ NCOMP) (64)	0.1	0.891	1.000
			0.2	0.888	1.000
		2NFC- NCOMP (40)	0.1	0.927	0.944
			0.2	0.926	0.860
		2(NFC- NCOMP) (32)	0.1	0.939	0.753
			0.2	0.939	0.634

Table 2A : 8 Wei in Series, RS = 0.9798 (Hi)
min λ = 0.001 f/hr, max λ = 0.008 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	2NFC (160)	0.1	0.9602	0.9859
			0.2	0.9506	0.9820
		2(NFC+ NCOMP) (176)	0.1	0.9565	0.9890
			0.2	0.9461	0.9870
		2NFC- NCOMP (152)	0.1	0.9619	0.9820
			0.2	0.9529	0.9809
		2(NFC- NCOMP) (144)	0.1	0.9638	0.9800
			0.2	0.9553	0.9770
2	Test 15 until 15 failed NFC=120	2NFC (240)	0.1	0.9706	0.9770
			0.2	0.9649	0.9719
		2(NFC+ NCOMP) (256)	0.1	0.9687	0.9820
			0.2	0.9627	0.9790
		2NFC- NCOMP (232)	0.1	0.9715	0.9730
			0.2	0.9659	0.9640
		2(NFC- NCOMP) (224)	0.1	0.9724	0.9650
			0.2	0.9671	0.9590
3	Test 15 until 11 failed NFC=88	2NFC (176)	0.1	0.9704	0.9719
			0.2	0.9637	0.9650
		2(NFC+ NCOMP) (192)	0.1	0.9679	0.9800
			0.2	0.9606	0.9760
		2NFC- NCOMP (168)	0.1	0.9716	0.9660
			0.2	0.9652	0.9590
		2(NFC- NCOMP) (160)	0.1	0.9729	0.9590
			0.2	0.9668	0.9510

Table 2A : 8 Wei in Series, RS = 0.9798 (Hi) (Cont...)
min λ = 0.001 f/hr, max λ = 0.008 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
4	Test 15 until 7 failed NFC=56	2NFC (112)	0.1	0.9764	0.9400
			0.2	0.9668	0.9280
		2(NFC+ NCOMP) (128)	0.1	0.9733	0.9579
			0.2	0.9624	0.9510
		2NFC- NCOMP (104)	0.1	0.9779	0.9240
			0.2	0.9691	0.9110
		2(NFC- NCOMP) (96)	0.1	0.9795	0.9080
			0.2	0.9713	0.8900
5	Test 15 until 3 failed NFC=24	2NFC (48)	0.1	0.9889	0.8000
			0.2	0.9814	0.7770
		2(NFC+ NCOMP) (64)	0.1	0.9857	0.8469
			0.2	0.9758	0.8360
		2NFC- NCOMP (40)	0.1	0.9906	0.7510
			0.2	0.9843	0.7280
		2(NFC- NCOMP) (32)	0.1	0.9922	0.6890
			0.2	0.9872	0.6530

Yee's results

Table 2A : 8 Wei in Series, RS = 0.980 (Hi)

min λ = 0.001 f/hr, max λ = 0.008 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 5 until 5 failed NFC=40	2NFC (80)	0.1	0.947	0.992
			0.2	0.930	0.989
		2(NFC+ NCOMP) (96)	0.1	0.937	0.994
			0.2	0.918	0.993
		2NFC- NCOMP (72)	0.1	0.951	0.989
			0.2	0.937	0.986
		2(NFC- NCOMP) (64)	0.1	0.956	0.985
			0.2	0.943	0.981
2	Test 15 until 15 failed NFC=120	2NFC (240)	0.1	0.978	0.918
			0.2	0.974	0.913
		2(NFC+ NCOMP) (256)	0.1	0.977	0.931
			0.2	0.972	0.924
		2NFC- NCOMP (232)	0.1	0.979	0.914
			0.2	0.975	0.901
		2(NFC- NCOMP) (224)	0.1	0.980	0.904
			0.2	0.975	0.889
3	Test 15 until 11 failed NFC=88	2NFC (176)	0.1	0.982	0.876
			0.2	0.977	0.860
		2(NFC+ NCOMP) (192)	0.1	0.980	0.894
			0.2	0.975	0.882
		2NFC- NCOMP (168)	0.1	0.983	0.861
			0.2	0.978	0.839
		2(NFC- NCOMP) (160)	0.1	0.983	0.840
			0.2	0.979	0.819

Yee's results

Table 2A : 8 Wei in Series, RS = 0.980 (Hi) (Cont...)
min λ = 0.001 f/hr, max λ = 0.008 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
4	Test 15 until 7 failed NFC=56	2NFC (112)	0.1	0.987	0.800
			0.2	0.981	0.779
		2(NFC+ NCOMP) (128)	0.1	0.985	0.839
			0.2	0.978	0.824
		2NFC- NCOMP (104)	0.1	0.988	0.776
			0.2	0.982	0.753
		2(NFC- NCOMP) (96)	0.1	0.989	0.746
			0.2	0.983	0.732
5	Test 15 until 3 failed NFC=24	2NFC (48)	0.1	0.994	0.621
			0.2	0.991	0.584
		2(NFC+ NCOMP) (64)	0.1	0.993	0.705
			0.2	0.988	0.685
		2NFC- NCOMP (40)	0.1	0.995	0.548
			0.2	0.992	0.514
		2(NFC- NCOMP) (32)	0.1	0.996	0.408
			0.2	0.993	0.417

Table 3A : 4 Exp and 4 Wei (mixed) in Series, RS = 0.9801 (Hi)
min λ = 0.0002 f/hr, max λ = 0.0008 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	2NFC (160)	0.1	0.9796	0.9190
			0.2	0.9791	0.8720
		2(NFC+ NCOMP) (176)	0.1	0.9777	0.9740
			0.2	0.9771	0.9489
		2NFC- NCOMP (152)	0.1	0.9805	0.8720
			0.2	0.9801	0.7980
		2(NFC- NCOMP) (144)	0.1	0.9815	0.7950
			0.2	0.9811	0.7000
2	Test 15 until 15 failed NFC=120	2NFC (240)	0.1	0.9802	0.8950
			0.2	0.9799	0.8250
		2(NFC+ NCOMP) (256)	0.1	0.9789	0.9510
			0.2	0.9786	0.9209
		2NFC- NCOMP (232)	0.1	0.9808	0.8370
			0.2	0.9805	0.7530
		2(NFC- NCOMP) (224)	0.1	0.9814	0.7670
			0.2	0.9812	0.6670
3	Test 15 until 11 failed NFC=88	2NFC (176)	0.1	0.9802	0.8920
			0.2	0.9797	0.8400
		2(NFC+ NCOMP) (192)	0.1	0.9785	0.9590
			0.2	0.9779	0.9320
		2NFC- NCOMP (168)	0.1	0.9810	0.8410
			0.2	0.9806	0.7550
		2(NFC- NCOMP) (160)	0.1	0.9819	0.7580
			0.2	0.9815	0.6590

Table 3A : 4 Exp and 4 Wei in Series, RS = 0.9801 (Hi) (Cont...)
min λ = 0.0002 f/hr, max λ = 0.0008 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
4	Test 15 until 7 failed NFC=56	2NFC (112)	0.1	0.9803	0.8950
			0.2	0.9797	0.8260
		2(NFC+ NCOMP) (128)	0.1	0.9777	0.9690
			0.2	0.9770	0.9439
		2NFC- NCOMP (104)	0.1	0.9815	0.8060
			0.2	0.9811	0.7300
		2(NFC- NCOMP) (96)	0.1	0.9828	0.7090
			0.2	0.9825	0.6000
5	Test 15 until 3 failed NFC=24	2NFC (48)	0.1	0.9815	0.8730
			0.2	0.9800	0.8020
		2(NFC+ NCOMP) (64)	0.1	0.9757	0.9740
			0.2	0.9739	0.9560
		2NFC- NCOMP (40)	0.1	0.9839	0.7250
			0.2	0.9831	0.6260
		2(NFC- NCOMP) (32)	0.1	0.9868	0.5150
			0.2	0.9862	0.4140

Yee's results

Table 3A : 4 Exp and 4 Wei (mixed) in Series, RS = 0.980 (Hi)
min λ = 0.002 f/hr, max λ = 0.008 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 5 until 5 failed NFC=40	2NFC (80)	0.1	0.979	0.942
			0.2	0.978	0.905
		2(NFC+ NCOMP) (96)	0.1	0.975	0.987
			0.2	0.974	0.976
		2NFC- NCOMP (72)	0.1	0.981	0.881
			0.2	0.980	0.805
		2(NFC- NCOMP) (64)	0.1	0.983	0.771
			0.2	0.982	0.684
2	Test 15 until 15 failed NFC=120	2NFC (240)	0.1	0.981	0.863
			0.2	0.980	0.800
		2(NFC+ NCOMP) (256)	0.1	0.979	0.941
			0.2	0.979	0.898
		2NFC- NCOMP (232)	0.1	0.981	0.881
			0.2	0.980	0.805
		2(NFC- NCOMP) (224)	0.1	0.982	0.725
			0.2	0.981	0.631
3	Test 15 until 11 failed NFC=88	2NFC (176)	0.1	0.981	0.864
			0.2	0.980	0.801
		2(NFC+ NCOMP) (192)	0.1	0.979	0.951
			0.2	0.978	0.907
		2NFC- NCOMP (168)	0.1	0.982	0.802
			0.2	0.981	0.698
		2(NFC- NCOMP) (160)	0.1	0.982	0.702
			0.2	0.982	0.591

Yee's results

Table 3A : 4 Exp and 4 Wei in Series, RS = 0.980 (Hi) (Cont...)
min λ = 0.002 f/hr, max λ = 0.008 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
4	Test 15 until 7 failed NFC=56	2NFC (112)	0.1	0.981	0.865
			0.2	0.980	0.787
		2(NFC+ NCOMP) (128)	0.1	0.978	0.952
			0.2	0.978	0.920
		2NFC- NCOMP (104)	0.1	0.982	0.769
			0.2	0.982	0.676
		2(NFC- NCOMP) (96)	0.1	0.983	0.644
			0.2	0.983	0.523
5	Test 15 until 3 failed NFC=24	2NFC (48)	0.1	0.982	0.843
			0.2	0.981	0.762
		2(NFC+ NCOMP) (64)	0.1	0.976	0.970
			0.2	0.975	0.941
		2NFC- NCOMP (40)	0.1	0.984	0.684
			0.2	0.984	0.580
		2(NFC- NCOMP) (32)	0.1	0.987	0.459
			0.2	0.987	0.386

Table 4A : 8 Exponential in Parallel, RS = 0.9345 (Hi)
min λ = 0.1000 f/hr, max λ = 0.1525 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	160	0.1	0.9306	0.9290
			0.2	0.9320	0.8180
2	Test 15 until 15 failed NFC=120	240	0.1	0.9325	0.9190
			0.2	0.9324	0.8210
3	Test 15 until 11 failed NFC=88	176	0.1	0.9309	0.9240
			0.2	0.9312	0.8329
4	Test 15 until 7 failed NFC=56	112	0.1	0.9307	0.9209
			0.2	0.9308	0.8310
5	Test 15 until 3 failed NFC=24	48	0.1	0.9280	0.9220
			0.2	0.9286	0.8370

Table 5A : 8 Wei in Parallel , RS = 0.9265 (Hi)
min λ = 0.100 f/hr, max λ = 0.128 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	161	0.1	0.9154	0.9540
			0.2	0.9157	0.8880
2	Test 15 until 15 failed NFC=120	241	0.1	0.9179	0.9510
			0.2	0.9170	0.8770
3	Test 15 until 11 failed NFC=88	177	0.1	0.9248	0.9180
			0.2	0.9250	0.8130
4	Test 15 until 7 failed NFC=56	113	0.1	0.9323	0.8630
			0.2	0.9318	0.7589
5	Test 15 until 3 failed NFC=24	51	0.1	0.9152	0.9270
			0.2	0.8980	0.8850

Table 6A : 4 EXP and 4 Wei (mixed) in Parallel, RS = 0.9408 (Hi)
min λ = 0.100 f/hr, max λ = 0.130 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	161	0.1	0.9342	0.9420
			0.2	0.9366	0.8500
2	Test 15 until 15 failed NFC=120	241	0.1	0.9362	0.9330
			0.2	0.9363	0.8510
3	Test 15 until 11 failed NFC=88	177	0.1	0.9389	0.9240
			0.2	0.9378	0.8310
4	Test 15 until 7 failed NFC=56	113	0.1	0.9416	0.8920
			0.2	0.9427	0.7819
5	Test 15 until 3 failed NFC=24	51	0.1	0.9383	0.9080
			0.2	0.9325	0.8410

Table 7A : 8 Exp in Series- Parallel, RS = 0.9249 (Hi)
min λ = 0.0003 f/hr, max λ = 0.0024 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	160	0.1	0.9241	0.9159
			0.2	0.9226	0.8620
2	Test 15 until 15 failed NFC=120	240	0.1	0.9253	0.8960
			0.2	0.9236	0.8360
3	Test 15 until 11 failed NFC=88	176	0.1	0.9252	0.8950
			0.2	0.9226	0.8390
4	Test 15 until 7 failed NFC=56	112	0.1	0.9222	0.9260
			0.2	0.9204	0.8789
5	Test 15 until 3 failed NFC=24	48	0.1	0.9153	0.9620
			0.2	0.9123	0.9260

Table 8A : 8 Wei in Series- Parallel, RS = 0.9328 (Hi)
min λ = 0.002 f/hr, max λ = 0.016 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	161	0.1	0.9106	0.9579
			0.2	0.8939	0.9510
2	Test 15 until 15 failed NFC=120	241	0.1	0.9247	0.9330
			0.2	0.9108	0.9230
3	Test 15 until 11 failed NFC=88	177	0.1	0.9266	0.9220
			0.2	0.9149	0.9119
4	Test 15 until 7 failed NFC=56	113	0.1	0.9414	0.8609
			0.2	0.9250	0.8419
5	Test 15 until 3 failed NFC=24	51	0.1	0.9699	0.7079
			0.2	0.9536	0.6740

Table 9A : 4 EXP and 4 Wei in Series- Parallel, RS = 0.9276 (Hi)
min λ = 0.005 f/hr, max λ = 0.020 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC = 80	161	0.1	0.9096	0.9400
			0.2	0.8904	0.9349
2	Test 15 until 15 failed NFC = 120	241	0.1	0.9238	0.9170
			0.2	0.9079	0.9040
3	Test 15 until 11 failed NFC = 88	177	0.1	0.9266	0.9050
			0.2	0.9109	0.8850
4	Test 15 until 7 failed NFC = 56	113	0.1	0.9393	0.8550
			0.2	0.9203	0.8390
5	Test 15 until 3 failed NFC = 24	51	0.1	0.9637	0.7430
			0.2	0.9451	0.7200

Table 10A :10 EXP and 5 Wei in Series- Parallel, RS = 0.9472 (Hi)
Exp : min λ = 0.025 f/hr, max λ = 0.075 f/hr.
Wei : min λ = 0.055 f/hr, max λ = 0.095 f/hr.
UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=150	301	0.1	0.9383	0.9710
			0.2	0.9369	0.9550
2	Test 20 until 20 failed NFC=300	601	0.1	0.9449	0.9370
			0.2	0.9432	0.8950
3	Test 20 until 15 failed NFC=225	451	0.1	0.9444	0.9349
			0.2	0.9430	0.8979
4	Test 20 until 10 failed NFC=150	301	0.1	0.9435	0.9290
			0.2	0.9415	0.9000
5	Test 20 until 5 failed NFC=75	152	0.1	0.9344	0.9809
			0.2	0.9310	0.9520

VI. CONCLUSIONS

The lower confidence limit procedures developed in this thesis are modifications and extensions of the lower confidence limit procedures written by Yee. Procedures were developed and evaluated in this thesis for more complex system structures than Yee examined. In addition a different method for estimating the parameters, in the Weibull distribution was used here rather than the maximum likelihood procedure used by Yee.

The evaluations show that the approximate lower confidence limit procedures are reasonably accurate for all system structures examined if the degrees of freedom are chosen judiciously. Although the equations given here for choosing an appropriate value for the degrees of freedom are for the system simulated in this thesis, it would be prudent to run simulations for complex systems that differ substantially from those examined in this thesis in order to determine an appropriate number for the degrees of freedom in the lower confidence limit equation.

The degrees of freedom equation derived in Appendix B, DF = 2NFC + 0.5(NC/NF), yielded lower confidence limit procedures with reasonable accuracy for test plans that were truncated on the number of failures.

VII. APPLICATION EXAMPLES

Based on the procedures evaluated by the RETP runs, three configurations of systems, a specific test plan and failure time data sets were constructed to illustrated the use of the procedures in providing a lower $100(1-\alpha)$ % confidence limit for system reliability. Actual results of one computer run for each case simulated are given below.

```
Case 1 : 8 Exponential components in Series
------- TEST PLAN 1 - Test 15 until 7 fails for each component
```

I. Raw Data

Comp i	Ordered Failure Times T(1)	T(2)	T(3)	T(4)	T(5)	T(6)	T(7)
1	695.0	1241.3	1365.7	2628.8	3304.9	3946.3	4014.2
2	142.5	212.7	230.8	315.2	401.4	1071.9	1222.0
3	111.9	394.7	422.6	506.5	519.5	558.5	582.6
4	325.5	356.2	441.7	837.8	844.2	873.7	894.4
5	62.6	110.0	124.3	126.8	325.8	384.0	502.8
6	20.0	85.6	107.0	108.0	161.5	190.1	201.6
7	34.1	77.9	100.4	142.1	156.0	180.9	193.5
8	6.1	34.6	48.1	65.3	95.2	136.6	160.4

II. Data Summary

Comp(i)	UT(i)	NC(i)	NF(i)	TT(i)	ELM(i)	ER(i)	ER(i)*TT(i)
1	10.0	15	7	49310.2	0.00012	0.03711	1829.9
2	10.0	15	7	13372.8	0.00045	0.13683	1829.9
3	10.0	15	7	7757.2	0.00077	0.23589	1829.9
4	10.0	15	7	11728.8	0.00051	0.15601	1829.9
5	10.0	15	7	5658.4	0.00106	0.32339	1829.9
6	10.0	15	7	2487.0	0.00241	0.73578	1829.9
7	10.0	15	7	2433.2	0.00247	0.75205	1829.9
8	10.0	15	7	1829.9	0.00328	1.00000	1829.9

III. Estimation Procedure for RSLOW

Parameter	df	Value

RS		0.93054	
ALPHA		0.1	
NFC		56	
CHISQD	104	122.86	(from the table Chi-square distribution)
LMU		0.00419	
RSLOW		0.92894	

CASE 4 : 8 Exponential components in parallel
------- TEST PLAN 1 - Test 15 until 7 fails for each component

I. Raw Data

Comp i	Ordered Failure Times T(1)	T(2)	T(3)	T(4)	T(5)	T(6)	T(7)
1	1.3899	2.4825	2.7315	5.2576	6.6099	7.8927	8.0285
2	0.5303	0.7915	0.8586	1.1727	1.4936	3.9886	4.5471
3	0.5839	2.0591	2.2047	2.6425	2.7106	2.9138	3.0398
4	2.1258	2.3264	2.8844	5.4715	5.5130	5.7055	5.8410
5	0.4813	0.8465	0.9563	0.9751	2.5065	2.9542	3.8674
6	0.1748	0.7471	0.9340	0.9424	1.4090	1.6593	1.7597
7	0.3296	0.7519	0.9697	1.3716	1.5065	1.7469	1.8685
8	0.0638	0.3635	0.5050	0.6849	0.9989	1.4334	1.6832

II. Data Summary

Comp(i)	UT(i)	NC(i)	NF(i)	TT(i)	ELM(i)	ER(i)	ER(i)*TT(i)
1	10.0	15	7	98.6204	0.06084	0.19467	19.1986
2	10.0	15	7	49.7592	0.12058	0.38583	19.1986
3	10.0	15	7	40.4724	0.14825	0.47436	19.1986
4	10.0	15	7	76.5961	0.07833	0.25065	19.1986
5	10.0	15	7	43.5263	0.13785	0.44108	19.1986
6	10.0	15	7	21.7044	0.27644	0.88455	19.1986
7	10.0	15	7	23.4926	0.25540	0.81722	19.1986
8	10.0	15	7	19.1986	0.31252	1.00000	19.1986

III. Estimation Procedure for RSLOW

Parameter	df	Value	
RS		0.93459	
ALPHA		0.1	
NFC		56	
CHISQD	112	131.56	(from the table Chi-square distribution)
LMU		0.42828	

RSLOW	0.93069

Case 7 : 8 Exponential components in Series-Parallel.
------- (4 components in parallel connected to 4 components in series)
------- TEST PLAN 1 - Test 15 until 7 fails for each component

I. Raw Data

Comp i	T(1)	T(2)	T(3)	T(4)	T(5)	T(6)	T(7)
			Ordered Failure Times				
1	463.309	827.505	910.497	1752.519	2203.295	2630.893	2676.161
2	95.019	141.817	153.837	210.102	267.609	714.622	814.687
3	74.604	263.110	281.713	337.648	346.349	372.313	388.416
4	217.008	237.491	294.454	558.552	562.782	582.436	596.273
5	41.716	73.360	82.882	84.511	217.230	256.028	335.172
6	13.354	57.073	71.346	71.990	107.635	126.749	134.425
7	22.755	51.917	66.954	94.703	104.022	120.616	129.016
8	4.053	23.095	32.090	43.517	63.475	91.080	106.955

II. Data Summary

Comp(i)	UT(i)	NC(i)	NF(i)	TT(i)	ELM(i)	ER(i)	ER(i)*TT(i)
1	10.0	15	7	32873.465	0.00018	0.03711	1219.9089
2	10.0	15	7	8915.184	0.00067	0.13683	1219.9089
3	10.0	15	7	5171.477	0.00116	0.23589	1219.9089
4	10.0	15	7	7819.181	0.00077	0.15601	1219.9089
5	10.0	15	7	3772.275	0.00159	0.32339	1219.9089
6	10.0	15	7	1657.971	0.00362	0.73578	1219.9089
7	10.0	15	7	1622.109	0.00370	0.75205	1219.9089
8	10.0	15	7	1219.909	0.00492	1.00000	1219.9089

III. Estimation Procedure for RSLOW

Parameter	df	Value	
RS		0.92496	
ALPHA		0.1	
NFC		56	
CHISQD	112	131.56	(from the table Chi-square distribution)
LMU		0.00674	
RSLOW		0.92223	

Case 2 : 8 Weibull components in Series
------- TEST PLAN 1 - Test 15 until 7 fails for each component

I. Raw Data

Comp i	T(1)	T(2)	T(3)	T(4)	T(5)	T(6)	T(7)
	Ordered Failure Times						
1	186.409	249.124	261.318	362.545	406.506	444.204	448.009
2	59.693	72.925	75.953	88.763	100.177	163.702	174.788
3	43.187	81.103	83.922	91.876	93.052	96.477	98.541
4	63.788	66.730	74.303	102.337	102.724	104.502	105.736
5	25.015	33.172	35.259	35.604	57.083	61.971	70.905
6	12.920	26.710	29.863	29.998	36.680	39.804	40.992
7	15.614	23.585	26.784	31.854	33.384	35.949	37.179
8	6.164	14.714	17.345	20.198	24.394	29.221	31.666

II. Data Summary

Comp(i)	UT(i)	NC(i)	NF(i)	TT(i)	ELM(i)	ER(i)	ER(i)*TT(i)
1	10.0	15	7	0.18E+09	0.40E-07	0.23E-04	4107.8930
2	10.0	15	7	0.43E+05	0.16E-03	0.96E-01	4107.8930
3	10.0	15	7	0.13E+11	0.56E-09	0.33E-06	4107.8930
4	10.0	15	7	0.69E+10	0.10E-08	0.60E-06	4107.8930
5	10.0	15	7	0.29E+05	0.24E-03	0.14E+00	4107.8930
6	10.0	15	7	0.25E+06	0.28E-04	0.17E-01	4107.8930
7	10.0	15	7	0.15E+07	0.45E-05	0.27E-02	4107.8930
8	10.0	15	7	0.41E+04	0.17E-02	0.10E+01	4107.8930

III. Estimation Procedure for RSLOW

Parameter	df	Value	
RS		0.92164	
ALPHA		0.1	
NFC		56	
CHISQD	112	131.56	(from the table Chi-square distribution)
LMU		0.00200	
RSLOW		0.91712	

IV. Workarea

Comp i	T'(1)	T'(2)	T'(3)	T'(4)	T'(5)	T'(6)	T'(7)
	Ordered Failure Times (h) raised to the power of Beta						
1	0.14E+07	0.30E+07	0.35E+07	0.84E+07	0.11E+08	0.14E+08	0.15E+08
2	0.68E+03	0.94E+03	0.10E+04	0.13E+04	0.16E+04	0.34E+04	0.38E+04
3	0.25E+08	0.42E+09	0.49E+09	0.74E+09	0.79E+09	0.93E+09	0.10E+10
4	0.64E+08	0.77E+08	0.12E+09	0.49E+09	0.50E+09	0.54E+09	0.57E+09
5	0.37E+03	0.62E+03	0.69E+03	0.71E+03	0.17E+04	0.20E+04	0.25E+04
6	0.94E+03	0.66E+04	0.89E+04	0.90E+04	0.15E+05	0.19E+05	0.21E+05
7	0.77E+04	0.30E+05	0.45E+05	0.79E+05	0.92E+05	0.12E+06	0.13E+06

8	0.22E+02	0.96E+02	0.13E+03	0.16E+03	0.23E+03	0.31E+03	0.35E+03

Case 5 : 8 Weibull components in parallel.
------- TEST PLAN 1 - Test 15 until 7 fails for each component

I. Raw Data

Comp i	T(1)	T(2)	T(3)	T(4)	T(5)	T(6)	T(7)
			Ordered Failure Times				
1	3.728	4.982	5.226	7.251	8.130	8.884	8.960
2	2.296	2.805	2.921	3.414	3.853	6.296	6.723
3	2.399	4.506	4.662	5.104	5.170	5.360	5.475
4	4.556	4.766	5.307	7.310	7.337	7.464	7.553
5	2.156	2.860	3.040	3.069	4.921	5.342	6.113
6	1.292	2.671	2.986	3.000	3.668	3.980	4.099
7	1.763	2.663	3.024	3.596	3.769	4.059	4.198
8	0.771	1.839	2.168	2.525	3.049	3.653	3.958

II. Data Summary

Comp(i)	UT(i)	NC(i)	NF(i)	TT(i)	ELM(i)	ER(i)	ER(i)*TT(i)
1	10.0	15	7	0.45E+04	0.16E-02	0.27E-01	120.5522
2	10.0	15	7	0.24E+03	0.30E-01	0.51E+00	120.5522
3	10.0	15	7	0.27E+05	0.26E-03	0.45E-02	120.5522
4	10.0	15	7	0.76E+05	0.92E-04	0.16E-02	120.5522
5	10.0	15	7	0.32E+03	0.22E-01	0.38E+00	120.5522
6	10.0	15	7	0.52E+03	0.13E-01	0.23E+00	120.5522
7	10.0	15	7	0.13E+04	0.55E-02	0.95E-01	120.5522
8	10.0	15	7	0.12E+03	0.58E-01	0.10E+01	120.5522

III. Estimation Procedure for RSLOW

Parameter	df	Value	
RS		0.92659	
ALPHA		0.1	
NFC		56	
CHISQD	116	135.89	(from the table Chi-square distribution)
LMU		0.07046	
RSLOW		0.92598	

IV. Workarea

Comp i	T'(1)	T'(2)	T'(3)	T'(4)	T'(5)	T'(6)	T'(7)
			Ordered Failure Times (h) raised to the power of Beta				

1	0.35E+02	0.77E+02	0.88E+02	0.21E+03	0.29E+03	0.37E+03	0.38E+03
2	0.38E+01	0.52E+01	0.55E+01	0.71E+01	0.86E+01	0.19E+02	0.21E+02
3	0.52E+02	0.90E+03	0.10E+04	0.16E+04	0.17E+04	0.20E+04	0.22E+04
4	0.70E+03	0.86E+03	0.14E+04	0.54E+04	0.55E+04	0.60E+04	0.63E+04
5	0.41E+01	0.69E+01	0.77E+01	0.78E+01	0.19E+02	0.22E+02	0.28E+02
6	0.20E+01	0.14E+02	0.19E+02	0.19E+02	0.32E+02	0.40E+02	0.44E+02
7	0.63E+01	0.24E+02	0.37E+02	0.65E+02	0.75E+02	0.96E+02	0.11E+03
8	0.64E+00	0.28E+01	0.37E+01	0.48E+01	0.66E+01	0.90E+01	0.10E+02

Case 8 : 8 Weibull components in series-parallel.
------- (4 components in parallel connected to 4 components in series)
------- TEST PLAN 1 - Test 15 until 7 fails for each component

I. Raw Data

Comp i	T(1)	T(2)	T(3)	T(4)	T(5)	T(6)	T(7)
1	5.142	6.872	7.209	10.001	11.214	12.254	12.359
2	3.184	3.889	4.051	4.734	5.343	8.731	9.322
3	3.344	6.279	6.497	7.113	7.204	7.469	7.629
4	6.379	6.673	7.430	10.234	10.272	10.450	10.574
5	83.383	110.574	117.532	118.681	190.276	206.571	236.351
6	25.840	53.419	59.727	59.996	73.360	79.608	81.983
7	24.289	36.688	41.663	49.551	51.931	55.920	57.835
8	8.219	19.619	23.126	26.931	32.526	38.962	42.221

(Header note: "Ordered Failure Times" spans T(1)–T(7).)

II. Data Summary

Comp(i)	UT(i)	NC(i)	NF(i)	TT(i)	ELM(i)	ER(i)	ER(i)*TT(i)
1	10.0	15	7	0.11E+05	0.66E-03	0.38E-01	399.5605
2	10.0	15	7	0.40E+03	0.18E-01	0.10E+01	399.5605
3	10.0	15	7	0.12E+06	0.58E-04	0.33E-02	399.5605
4	10.0	15	7	0.33E+06	0.21E-04	0.12E-02	399.5605
5	10.0	15	7	0.26E+06	0.27E-04	0.15E-02	399.5605
6	10.0	15	7	0.16E+07	0.44E-05	0.25E-03	399.5605
7	10.0	15	7	0.65E+07	0.11E-05	0.61E-04	399.5605
8	10.0	15	7	0.67E+04	0.10E-02	0.60E-01	399.5605

III. Estimation Procedure for RSLOW

Parameter	df	Value	
RS		0.93682	
ALPHA		0.1	
NFC		56	
CHISQD	104	122.86	(from the table Chi-square distribution)
LMU		0.01922	

RSLOW 0.93382

IV. Workarea

Comp i	T'(1)	T'(2)	T'(3)	T'(4)	T'(5)	T'(6)	T'(7)
	Ordered Failure Times (h) raised to the power of Beta						
1	0.84E+02	0.18E+03	0.21E+03	0.51E+03	0.69E+03	0.88E+03	0.90E+03
2	0.63E+01	0.87E+01	0.93E+01	0.12E+02	0.14E+02	0.32E+02	0.35E+02
3	0.23E+03	0.40E+04	0.47E+04	0.71E+04	0.75E+04	0.88E+04	0.97E+04
4	0.30E+04	0.37E+04	0.58E+04	0.23E+05	0.24E+05	0.26E+05	0.27E+05
5	0.34E+04	0.57E+04	0.63E+04	0.65E+04	0.15E+05	0.18E+05	0.23E+05
6	0.60E+04	0.42E+05	0.57E+05	0.58E+05	0.99E+05	0.12E+06	0.13E+06
7	0.33E+05	0.13E+06	0.19E+06	0.33E+06	0.39E+06	0.49E+06	0.55E+06
8	0.36E+02	0.16E+03	0.21E+03	0.27E+03	0.37E+03	0.50E+03	0.57E+03

APPENDIX A. DERIVATION OF FORMULA FOR BIAS VALUE

Suppose Y has an extreme-value distribution; i.e., $Y \sim EV(\sigma, \mu)$.

Suppose $Y_{(r+1)} \leq Y_{(r+2)} \leq Y_{(r+3)} \leq \ldots \leq Y_{(n-s)}$ are the middle n-s ordered statistics in a random sample of size n from extreme-value distribution with pdf. and Cdf. given by

$$f(y; \mu, \sigma) = \frac{1}{\sigma} e^{(y-\mu)/\sigma} \exp\{-e^{(y-\mu)/\sigma}\}, \qquad \begin{array}{r} -\infty < y < \infty \\ -\infty < \mu < \infty \\ \sigma > 0 \end{array} \tag{A.1}$$

$$F(y; \mu, \sigma) = 1 - \exp\{-e^{(y-\mu)/\sigma}\} \tag{A.2}$$

where $\sigma = 1/\beta$, $\mu = \ln\ \theta$ and ln is the natural logarithm.

Letting $L^{\times}$ denote the approximated likelihood function and $X_{(i)} \equiv (Y_{(i)} - \mu)/\sigma$, from Chapter II the approximated likelihood equations for μ and σ are :

$$\frac{\delta \ln L}{\delta \mu} \approx \frac{\delta \ln L^{\times}}{\delta \mu} = -\frac{1}{\sigma}[r(\gamma - \delta X_{(r+1)}) - \tag{A.3}$$

$$s(1 - \alpha_{n-s} + \beta_{n-s} X_{(n-s)}) + \sum_{i=r+1}^{n-s} (\alpha_i - \beta_i X_{(i)})] = 0$$

$$\frac{\delta \ln L}{\delta\sigma} \approx \frac{\delta \ln L^{\times}}{\delta\sigma} = -\frac{1}{\sigma}[A + rX_{(r+1)}(\gamma - \delta X_{(r+1)}) - \tag{A.4}$$

$$sX_{(n-s)}(1 - \alpha_{n-s} + \beta_{n-s}X_{(n-s)}) + \sum_{i=r+1}^{n-s} X_{(i)}(\alpha_i - \beta_i X_{(i)})] = 0.$$

It is not possible to compute the conditional s-bias of $\hat{\sigma}$ exactly nor the conditional s-bias of $\hat{\mu}$. The conditional s-bias of $\hat{\sigma}$ however, can be approximated by :

$$\frac{E\left[\dfrac{\delta \ln L^{\times}}{\delta\sigma}\right]}{E\left[-\dfrac{\delta^2 \ln L^{\times}}{\delta\sigma^2}\right]} \tag{A.5}$$

Working out this approximation is tedious, but Balakrishnan and Varadan [Ref. 6 p-149] provide a table of constants for the conditional s-bias of $\hat{\sigma}$.

Let C denote the bias value of $\hat{\sigma}$ from the table. Then

$$\frac{E[\hat{\sigma}] - \sigma}{\sigma} = C$$

$$E[\hat{\sigma}] = \sigma + \sigma C = (1 + C)\sigma.$$

Thus $\hat{\sigma}^{\times} = \hat{\sigma}/(1 + C)$ is nearly unbiased estimator of σ.

APPENDIX B. DERIVATION OF EQUATION FOR DEGREES OF FREEDOM

One basic expression used to determine the degrees of freedom, DF, for the chi-square distribution when obtaining confidence limits on the reliability of a complex system is DF = 2NFC, where NFC is total number of failed components.

In chapter V, the analysis of the simulation results show that another formulae for computing degrees of freedom yielded a more accurate lower confidence limit procedure. The formula DF = 2NFC - NCOMP, for example, gave more accurate results than procedures that used DF = 2NFC. Moreover, the accuracy associated with any fixed formula for DF degraded as the extent of truncation increased. That is, if for a particular component, NC items are tested until NF fail, the accuracy of each procedure decreased as NF decreased. The decrease in accuracy became significant for values of NF < NC/2. Consequently, a formula for DF was developed that included NC/NF as one of the terms in the expression for DF.

This formula for DF has the form DF = 2NFC $\pm$ c(NC/NF) where NF is the smallest failure truncation value for each component type, NC is the number of items placed on test for that component with smallest NF and c is an unknown constant. An equation was established using this formula for each of the series system cases simulated using the value of DF that yielded the most accurate result and the appropriate values of NFC, NC and NF for that case. Each equation can be solved for c. Since there were thirty series system cases simulated (See Table 1A through

Table 3B), the resulting thirty equations yielded thirty values of c. An averaging process using least square methods was used to determine one value for c. The following set of equations show these computations for each confidence level value.

A. ALPHA = 0.1 (CONFIDENCE 90 %)

$152 = 160 + c$ reduces to $-8 = c$

$232 = 240 + c$ reduces to $-8 = c$

$168 = 176 + 1.36c$ reduces to $-8 = 1.36c$

$104 = 112 + 2.14c$ reduces to $-8 = 2.14c$

$40 = 48 + 5.0c$ reduces to $-8 = 5.0c$

$152 = 160 + c$ reduces to $-8 = c$

$232 = 240 + c$ reduces to $-8 = c$

$168 = 176 + 1.36c$ reduces to $-8 = 1.36c$

$104 = 112 + 2.14c$ reduces to $-8 = 2.14c$

$40 = 48 + 5.0c$ reduces to $-8 = 5.0c$

$144 = 160 + c$ reduces to $-16 = c$

$224 = 240 + c$ reduces to $-16 = c$

$160 = 176 + 1.36c$ reduces to $-16 = 1.36c$

$96 = 112 + 2.14c$ reduces to $-16 = 2.14c$

$64 = 48 + 5.0c$ reduces to $16 = 5.0c$

$144 = 160 + c$ reduces to $-16 = c$

$224 = 240 + c$ reduces to $-16 = c$

$160 = 176 + 1.36c$ reduces to $-16 = 1.36c$

$112 = 112 + 2.14c$ reduces to $0 = 2.14c$

$64 = 48 + 5.0c$ reduces to $16 = 5.0c$

$160 = 160 + c$ reduces to $0 = c$

$256 = 240 + c$ reduces to $16 = c$

$192 = 176 + 1.36c$ reduces to $16 = 1.36c$

$128 = 112 + 2.14c$ reduces to $16 = 2.14c$

$64 = 48 + 5.0c$ reduces to $16 = 5.0c$

$152 = 160 + c$ reduces to $-8 = c$

$232 = 240 + c$ reduces to $-8 = c$

$176 = 176 + 1.36c$ reduces to $0 = 1.36c$

$112 = 112 + 2.14c$ reduces to $0 = 2.14c$

$48 = 48 + 5.0c$ reduces to $0 = 5.0c$

The above equations form a linear system that is overdetermined. By using the normal equation, $A^TA\,x = A^Tb$, we then find the least square solution to that overdetermined system which yields $c = 1.1$. Replacing the constants in the general formula, we have the approximating equation for obtaining the degrees of freedom ;

$DF = 2NFC + 1.1(NC/NF)$

B. ALPHA = 0.2 (CONFIDENCE 80 %)

$152 = 160 + c$ reduces to $-8 = c$

$232 = 240 + c$ reduces to $-8 = c$

$168 = 176 + 1.36c$ reduces to $-8 = 1.36c$

$104 = 112 + 2.14c$ reduces to $-8 = 2.14c$

$40 = 48 + 5.0c$ reduces to $-8 = 5.0c$

$152 = 160 + c$ reduces to $-8 = c$

$232 = 240 + c$ reduces to $-8 = c$

$168 = 176 + 1.36c$ reduces to $-8 = 1.36c$

$104 = 112 + 2.14c$ reduces to $-8 = 2.14c$

$40 = 48 + 5.0c$ reduces to $-8 = 5.0c$

$144 = 160 + c$ reduces to $-16 = c$

$224 = 240 + c$ reduces to $-16 = c$

160 = 176 + 1.36c reduces to -16 = 1.36c

96 = 112 + 2.14c reduces to -16 = 2.14c

64 = 48 + 5.0c reduces to 16 = 5.0c

144 = 160 + c reduces to -16 = c

224 = 240 + c reduces to -16 = c

160 = 176 + 1.36c reduces to -16 = 1.36c

96 = 112 + 2.14c reduces to -16 = 2.14c

64 = 48 + 5.0c reduces to 16 = 5.0c

152 = 160 + c reduces to -8 = c

240 = 240 + c reduces to 0 = c

176 = 176 + 1.36c reduces to 0 = 1.36c

112 = 112 + 2.14c reduces to 0 = 2.14c

48 = 48 + 5.0c reduces to 0 = 5.0c

152 = 160 + c reduces to -8 = c

232 = 240 + c reduces to -8 = c

168 = 176 + 1.36c reduces to -8 = 1.36c

112 = 112 + 2.14c reduces to 0 = 2.14c

48 = 48 + 5.0c reduces to 0 = 5.0c

Using the normal equation, $A^T A x = A^T b$ again, the least squares solution is c = 1.0 . The approximate equation for obtaining the degrees of freedom is DF = 2NFC + (NC/NF)

APPENDIX C. USERS' GUIDE FOR RETP

Reliability Estimation Test Plan (RETP).

1. Brief Description

   RETP is a computer program written in FORTRAN that runs on the Amdahl mainframe at NPGS. It allows the user to simulate exponential Weibull failure times of component items being tested to evaluate the accuracy of a confidence limit estimation procedure based on Type II data censoring (that is, testing n_i items of component i until f_i of them fail).

2. Program Input. (INI.DATA)

   The inputs of the program are specified to the program via an input file called INI.DAT. A sample input file is shown below.

   This *sample input* refers to a system with configuration as represented by Figure 4.4.

```
c==========================================================================
c--------------------------------------------------------------------------
This file contains the inputs required by the RETP model.
Update only the numerical values between dotted lines as appropriate.
Do not delete any of the comment lines. (INI.DAT)
c-------------------------------------------------------------------------
c Value     Type   Units   Description                          Variable
c-------------------------------------------------------------------------
  3         INT      -     Configuration of the system          CONFIG
                             1 = all subsystems in series
                             2 = all subsystems in parallel
                             3 = subsystems are connected
                                 in series-parallel
  16807.0   REAL     -     Initial random seed                  ISEED
  15        INT      -     Total # of subsystems in system      NCOMP
  10        INT      -     # of exponential subsystems          NEXP
  5         INT      -     # of Weibull subsystems              NWEI
  0         INT      -     # of geometric subsystems            NGEO
  0.10      REAL     -     Desired significance level           ALPHA
  1000      INT      -     # of replications desired            NREP
  3         INT      -     Test case number                     TCN
                             1 = all exponential
                             2 = all Weibull
                             3 = EXP + WEI
                             4 = EXP + WEI + GEO
  15        INT      -     Number of cut sets                   NCS
c-------------------------------------------------------------------------
```

```
c---------------------------------------------------------------------------
c TEST PLAN : Testing NC(I) items of component i until NF(i) of them
c             fail.
c             (Use REAL numbers ONLY !!!)
c---------------------------------------------------------------------------
c                      Components Configuration
c CSERIES(i)      CSSGROUP(i)          CSGROUP(i)           NCGROUP(i)
c---------------------------------------------------------------------------
     2                4                    8                    14
     2                4                    8                    14
     2                4                    8                    14
     2                4                    8                    14
     2                4                    8                    14
     2                4                    8                    14
     2                4                    8                    14
     2                4                    8                    14
     2                4                    6                    14
     2                4                    6                    14
     2                4                    6                    14
     2                4                    6                    14
     1                1                    6                    14
     1                1                    6                    14
     1                1                    1                     1
c---------------------------------------------------------------------------
c  Comp   Comp      Comp Parameters          Util           Test Plan
c Number  Type     Scale      Shape       Time/Cycle          input
c   I     TY(I)   PARM(1,I)  PARM(2,I)    UT(I)  UC(I)     NC(I)  NF(I)
c  Int     Int      Real       Real       (hrs)   Int       Int    Int
c---------------------------------------------------------------------------
   1.0    2.0     0.07500     1.5        10.0               20.0   15.0
   2.0    2.0     0.05500     2.0         8.0               20.0   15.0
   3.0    2.0     0.17500     1.5         8.0               20.0   15.0
   4.0    2.0     0.09500     3.0         2.0               20.0   15.0
   5.0    2.0     0.12500     2.0         5.0               20.0   15.0
   6.0    1.0     0.05500     1.0         1.0               20.0   15.0
   7.0    1.0     0.15000     1.0         1.0               20.0   15.0
   8.0    1.0     0.15500     1.0         2.0               20.0   15.0
   9.0    1.0     0.07500     1.0         2.0               20.0   15.0
  10.0    1.0     0.05000     1.0         2.0               20.0   15.0
  11.0    1.0     0.17500     1.0         2.0               20.0   15.0
  12.0    1.0     0.08500     1.0         1.5               20.0   15.0
  13.0    1.0     0.17500     1.0         1.0               20.0   15.0
  14.0    1.0     0.05000     1.0         1.0               20.0   15.0
  15.0    1.0     0.02500     1.0         1.0               20.0   15.0
c---------------------------------------------------------------------------
c  Note : TY(I)=1 EXPONENTIAL  P(surv) = exp(-PARM(1,I) * T)
c         TY(I)=2 WEIBULL      P(surv) = exp(-(PARM(1,I)*T)**PARM(2,I))
c         TY(I)=3 GEOMETRIC    P(surv) = PARM(1,I) ** T
c---------------------------------------------------------------------------
c SYSTEM CONFIGURATION : Identification of CUT SETS
c                        Minimum groups of subsystems that have to fail
c                        for the system to fail.
c---------------------------------------------------------------------------
c Cut Set   # in Set   List of Components in Cutset
c   J       COMP(J,1)  COMP(J,2) .....up to COMP(J,1) components
c---------------------------------------------------------------------------
```

```
1         1    1   0   0   0   0   0   0   0   0
2         1    2   0   0   0   0   0   0   0   0
3         1    3   0   0   0   0   0   0   0   0
4         1    4   0   0   0   0   0   0   0   0
5         1    5   0   0   0   0   0   0   0   0
6         1    6   0   0   0   0   0   0   0   0
7         1    7   0   0   0   0   0   0   0   0
8         1    8   0   0   0   0   0   0   0   0
9         1    9   0   0   0   0   0   0   0   0
10        1   10   0   0   0   0   0   0   0   0
11        1   11   0   0   0   0   0   0   0   0
12        1   12   0   0   0   0   0   0   0   0
13        1   13   0   0   0   0   0   0   0   0
14        1   14   0   0   0   0   0   0   0   0
15        1   15   0   0   0   0   0   0   0   0
c------------------------------------------------------------
```

3. Program Flow and Logic. (NAME.DEF,PARM.DEF and RETP.FOR)

Input parameters are first read in by the program by calling the INPUT subroutine. The program then evokes the SIM subroutine which generates the random failure times and computes the key statistics required in the procedure. The next subroutine EVAL determines the measures of accuracy for each case. REPORT is the subroutine which generates the output file for the run OUT.DAT.

The variables in the program RETP.FOR are described in the file NAME.DEF as listed below.

```
c-----------------------------------------------------------------
c  This file contains the declaration for input and output variables
c  used in the RETP model. (NAME.DEF)
c-----------------------------------------------------------------
c  Input Variables.
c  ---------------
c  ISEED        = initial random seed selected. (Real)
c  SEED         = current random seed (Real)
c  RS           = true overall system reliability.
c  ALPHA        = desired significance level (Real)
c  NREP         = number of replication desired for simulation (Integer)
c  TPN          = test plan number
c  TCN          = test case number (1, 2, 3 or 4)
c  NCOMP        = total # of components in the system (Integer)
c  NEXP         = number of components with EXP failure times.(Integer)
c  NWEI         = number of components with WEI failure times.(Integer)
c  NGEO         = number of components with GEO failure times. (Integer)
c  Distribution:    EXPonential        WEibull        GEOmetric
c  TY(i) = type:         1                 2              3
c   PARM(1,i)  :    Scale(1/hr)        Scale(1/hr)       Prob
c   PARM(2,i)  :         -               Shape             -
c  UT(i)        = utilization time (hrs) for component i (EXP and WEI)
c  UC(i)        = utilization cycles for component i (GEO only)
c  NC(i)        = number of test samples (sample size) for component i.
c  NF(i)        = desired number of failures in test for component i.
```

```
c  NCS         = number of cut sets for the system.
c  COMP(J,K)   = kth parameter of cutset j ( first being the number of
c                components belonging to the cut-set)
c  CONFIG      = configuration number of components arrangement(integer).
c  CSERIES(i) = number of subsystems in series in a subsubgroup(integer)
c  CSGROUP(i) = number of subsystems i in a subgroup (integer)
c  CSSGROUP(i)= number of subsystems i in a subsubgroup (integer)
c  NCGROUP(i) = number of subsystems i in a group (integer)
c
c  Assumed Variables.
c  -----------------
c  MAXCOMP     = maximum number of components allowed in the system
c  MAXREP      = maximum number of replication permitted.
c  MAXCUT      = maximum number of cut-sets.
c
c  Program and Output Variables.
c  ----------------------------
c  TT(i)      = total accumulated failure time (in hour) for component i
c               (EXP and WEI only)
c  TC(i)      = total accumulated cycles to failure (incl. failure cycle)
c               for component i (GEO only)
c  EBETA(i)   = estimate for shape parameter of component i (if Weibull)
c  REL1(j)    = actual reliability for cut-set j.
c  REL2(j)    = computed reliability for cut-set j for current replication
c  ELM(i)     = estimated component failure rate (1/hrs) for component i
c  ELMAX(m)   = max. estimated component failure rate for rep. m (1/hrs)
c  ER(i)      = ratio of estimated failure rate to ELMAX.
c  NFC(m)     = total number of failed test components.
c  LMU(m)     = upper confidence limit for failure rate (1/hrs).
c  RSL(m)     = lower confidence limit estimated for system reliability
c               for the mth replication.
c  ORSL(m)    = ascending order of RSL(M).
c  RSLOW      = (1-ALPHA)x100 percentile of set of RSL(M).
c  LEVEL      = achieved confidence interval i.e proportion of RSL(M)
c               that are lesser than RS (conservative estimate)
c
c-- END OF NAME.DEF ------------------------------------------------
c
```

Together with the main program in RETP.FOR are the other subroutines needed in the simulation. The declaration of variables is done in the file PARM.DEF. Relevant descriptions are included as comment lines in the source code to help explain the program segments. A listing of PARM.DEF and RETP.FOR is given below.

```
c------------------------------------------------------------------
c------------------------------------------------------------------
c  This file contains the declaration for input and output variables
c  used in the RETP model.   (PARM.DEF)
c------------------------------------------------------------------
c
      INTEGER  MAXCOMP, MAXREP
      PARAMETER( MAXCOMP = 100, MAXREP = 1000, MAXCUT = 20 )
```

```
      REAL*8 ISEED, SEED
      INTEGER NREP, TCN, NCOMP, NEXP, NWEI, NGEO, NCS,
     +        NC(MAXCOMP),NF(MAXCOMP), TY(MAXCOMP), NFC(MAXREP),
     +        UC(MAXCOMP), TC(MAXCOMP), COMP(MAXCUT,MAXCOMP),
     +        CONFIG, CSERIES(MAXCOMP), CSSGROUP(MAXCOMP),
     +        CSGROUP(MAXCOMP),NCGROUP(MAXCOMP),DFR
      REAL*8  RS, ALPHA, UT(MAXCOMP), TT(MAXCOMP),
     +        PARM(2,MAXCOMP), ELM(MAXCOMP), ER(MAXCOMP),
     +        LMU(MAXREP), RSL(MAXREP), ORSL(MAXREP),
     +        ELMAX(MAXREP), RSLOW, LEVEL, EBETA(MAXCOMP),
     +        REL1(MAXCUT), REL2(MAXCUT)
c
      COMMON/BLOCK1/ISEED, SEED, NREP, TCN, NCOMP, NC, NF, NEXP, NWEI,
     +              NGEO, NCS, TY, NFC, UC, TC, COMP, CONFIG, CSERIES,
     +              CSSGROUP, CSGROUP, NCGROUP, DFR
      COMMON/BLOCK2/RS, ALPHA, UT, TT, PARM, ELM, ER, LMU,
     +              RSL, ORSL, ELMAX, RSLOW, LEVEL, EBETA, REL1, REL2
c
c  END OF PARM.DEF ---------------------------------------------------
c
c--------------------------------------------------------------------
c  This file contains the main program and the subroutines
c  for the Reliability Estimation Test Plan (RETP) model.
c
c  IBM Mainframe version.
c  Test Plan 1 : Testing NC(I) items for component i
c
c--------------------------------------------------------------------
c  1.  Main Program (RETP).
c-------------------------------------------------------------------
      PROGRAM RETP
c
c  Include the declaration files.
c
      INCLUDE 'NAME DEF'
      INCLUDE 'PARM DEF'
c
c  Read the input data.
c
      CALL INPUT
c
c  Activate simulation
c
      CALL SIM
c
c  Process and evaluate output data.
c
      CALL EVAL
c
c  Generate simulation report.
c
      CALL REPORT
c
      STOP
      END
c-------------------------------------------------------------------
```

```
c 2. Input Initialisation Subroutine (INPUT).
c-----------------------------------------------------------------------
      SUBROUTINE INPUT
c
c  This subroutine reads in the inputs for the RETP model.
c
c  Include the declaration file.
c
      INCLUDE 'PARM DEF'
c
      INTEGER I, J, K, DUM2(11), DUM3(4)
      REAL*8 DUM1(7)
c
c  Read data from file 'INI.DAT' designated as logic unit 1.
c
      OPEN (UNIT=1,FILE='/INI data A1')
c
      READ(1,*) CONFIG
      READ(1,*) ISEED
      READ(1,*) NCOMP
      READ(1,*) NEXP
      READ(1,*) NWEI
      READ(1,*) NGEO
      READ(1,*) ALPHA
      READ(1,*) NREP
      READ(1,*) TCN
      READ(1,*) NCS
c
      DO 40 I = 1, NCOMP
         READ(1,*) DUM3
           CSERIES(I)  = DUM3(1)
           CSSGROUP(I) = DUM3(2)
           CSGROUP(I)  = DUM3(3)
           NCGROUP(I)  = DUM3(4)
  40  CONTINUE
c
      DO 50 K = 1, NCOMP
         READ(1,*) DUM1
c
         I = NINT(DUM1(1))
         TY(I) = NINT(DUM1(2))
c
         IF (TY(I).EQ.1) THEN
            PARM(1,I) = DUM1(3)
            PARM(2,I) = DUM1(4)
            UT(I) = DUM1(5)
            NC(I) = NINT(DUM1(6))
            NF(I) = NINT(DUM1(7))
            EBETA(I) = 0
         ELSEIF (TY(I).EQ.2) THEN
            PARM(1,I) = DUM1(3)
            PARM(2,I) = DUM1(4)
            UT(I) = DUM1(5)
            NC(I) = NINT(DUM1(6))
            NF(I) = NINT(DUM1(7))
         ELSEIF (TY(I).EQ.3) THEN
```

```
             PARM(1,I) = DUM1(3)
             PARM(2,I) = DUM1(4)
             UC(I) = NINT(DUM1(5))
             UT(I) = DUM1(5)
             NC(I) = NINT(DUM1(6))
             NF(I) = NINT(DUM1(7))
          ENDIF
   50  CONTINUE
c
       DO 80 I = 1, NCS
          READ(1,*) DUM2
          J = DUM2(1)
          COMP(J,1) = DUM2(2)
          DO 70 K=1,COMP(J,1)
             COMP(J,K+1) = DUM2(K+2)
   70     CONTINUE
   80  CONTINUE
       CLOSE(UNIT=1)
       RETURN
       END
c-----------------------------------------------------------------------------
c 3. Subroutine for Simulation.
c-----------------------------------------------------------------------------
       SUBROUTINE SIM
c
c  This subroutine simulates NREP possible outcomes of the test plan
c  desired in order to obtain the raw estimates of LMU(M) and RSL(M)
c  for each of the replication.
c
c  Include the declaration file
c  and declare local variables.
c
       INCLUDE 'PARM DEF'
c
       INTEGER I, J, K, M, ISUM, KEY, L, C, CC, CCC
       REAL*8 UNI
       REAL*8 SUM, PROD, RP, RPP, RSP, RSS,
      +       FT(MAXCOMP), OFT(MAXCOMP),YY(MAXCOMP)
c
       LOGICAL TEMP
c
       SEED = ISEED
c
c  Compute overall true system reliability RS.
c
       C = 0
       CC = 0
       CCC = 0
       RP = 0.0
       RPP = 1.0
       RSP = 0.0
       RSS= 1.0
       RS = 1.0
       J = 1
       TEMP = .FALSE.
       DO WHILE(.NOT.TEMP)
```

```
         DO 20 L = J, J + (CSERIES(J) - 1)
            PROD = 1.0
            DO 10 I = 1, COMP(L,1)
               K = COMP(L,I+1)
               PROD = PROD*(1- SURV(TY(K),PARM(1,K),PARM(2,K),UT(K)))
   10       CONTINUE
            REL1(L) = 1.0 - PROD
            RSS = RSS * REL1(L)
   20    CONTINUE
         L = L - 1
         RP = 1.0 - (1.0 -RSS)*(1.0 -RP)
         RSS =1.0
         IF (L.GE.(C + CSSGROUP(L))) THEN
            RPP = RPP * RP
            RP = 0.0
            C = C + CSSGROUP(L)
         ENDIF
         IF (L.GE.(CC + CSGROUP(L))) THEN
            RSP = 1.0 - (1.0 - RPP)*(1.0 - RSP)
            RPP = 1.0
            CC = CC + CSGROUP(L)
         ENDIF
         IF (L.GE.(CCC + NCGROUP(L))) THEN
            RS = RS * RSP
            RSP = 0.0
            CCC = CCC + NCGROUP(L)
         ENDIF
         J = J + CSERIES(L)
         IF (L.GE.NCOMP) TEMP = .TRUE.
      ENDDO
c
c  Start of Simulation
c  (Initialize replication counter M)
c
      M = 1
      DO WHILE (M.LE.NREP)
c
c  Test Plan : Sample and determine unknown TT(I)
c  ---------   with known NC(I) until NF(I) fails.
c
c  Generate NC(I) failure times, put them in ascending order
c  with the smallest failure time on the top of the list.
c
         DO 70 I = 1, NCOMP
c
            DO 40 K = 1, NC(I)
               CALL LRNDPC(SEED,UNI,1)
               IF (TY(I).EQ.1) THEN
                 FT(K)= -LOG(UNI)/PARM(1,I)
               ELSEIF (TY(I).EQ.2) THEN
                 FT(K)=(1.0/PARM(1,I))*(-LOG(UNI))**(1.0/PARM(2,I))
               ELSEIF (TY(I).EQ.3) THEN
                 FT(K)= 1.0
                 DO WHILE (UNI.LT.PARM(1,I))
                    FT(K) = FT(K) + 1.0
                    CALL LRNDPC(SEED,UNI,1)
```

```
                  ENDDO
                ENDIF
 40           CONTINUE
c
c  Buble Sort the failure times in ascending order.
c
      CALL BUBBLE(NC(I),FT,OFT)
c
c  Take logarithm of the ordered failure times.
c
      DO 45 K=1, NC(I)
         YY(K) = LOG(OFT(K))
  45  CONTINUE
c
c  Compute the total time accumulated in the test and the estimate
c  for the failure rate of the component as in the procedure.
c
c
       IF (TY(I).NE.2) THEN
         SUM = 0.0
         DO 50 K = 1, NF(I)
            SUM = SUM + OFT(K)
  50     CONTINUE
         TT(I) = FLOAT(NC(I)-NF(I))*OFT(NF(I)) + SUM
         IF (TY(I).EQ.1) THEN
            ELM(I) = FLOAT(NF(I) - 1)/TT(I)
         ELSEIF (TY(I).EQ.3) THEN
            ELM(I) = FLOAT(NF(I) - 1)/TT(I)
         ENDIF
       ELSEIF (TY(I).EQ.2) THEN
c
         CALL APROXMLE(YY,NC(I),NF(I),EBETA(I))
c
         EBETA(I) = BN(NC(I))*EBETA(I)
c
         SUM = 0.0
         DO 60 K = 1, NF(I)
            SUM = SUM + OFT(K)**EBETA(I)
  60     CONTINUE
c
         TT(I) = FLOAT(NC(I)-NF(I))*OFT(NF(I))**EBETA(I) + SUM
         ELM(I) = FLOAT(NF(I))/TT(I)
       ENDIF
c
  70  CONTINUE
c
c  Determine the total number of failed test items.
c
      ISUM = 0
      DO 80 I = 1, NCOMP
         ISUM = ISUM + NF(I)
  80  CONTINUE
      NFC(M) = ISUM
c
c  Determine the maximum failure rate estimate
```

```
c  and identify the component.
c
      ELMAX(M) = 0.0
      KEY = 0
      DO 90 I = 1, NCOMP
         IF (ELM(I).GT.ELMAX(M)) THEN
            ELMAX(M) = ELM(I)
            KEY = I
         ENDIF
   90 CONTINUE
c
c  Compute the ratios of the failure rate estimate to their maximum
c
      DO 100 I = 1, NCOMP
         ER(I) = ELM(I)/ELMAX(M)
  100 CONTINUE
c
c  Determine LMU(M)
c
      SUM = 0.0
      DO 110 I = 1, NCOMP
         SUM = SUM + (ER(I)*TT(I))
  110 CONTINUE
c
      DFR = 2*NFC(M)-2*NCOMP
      LMU(M) = CHISQD(1-ALPHA,DFR)/(2*SUM)
c
c  Compute estimate of overall reliability RSL(M) for the system.
c
      C = 0
      CC = 0
      CCC = 0
      RP = 0.0
      RPP = 1.0
      RSP = 0.0
      RSS= 1.0
      RSL(M) = 1.0
      J = 1
      TEMP = .FALSE.
      DO WHILE(.NOT.TEMP)
         DO 120 L = J, J + (CSERIES(J) - 1)
            PROD = 1.0
            DO 115 I = 1, COMP(L,1)
               K = COMP(L,I+1)
               IF (TY(K).EQ.1) THEN
                 PROD = PROD*(1-SURV(TY(K),LMU(M)*ER(K),EBETA(K),UT(K)))
               ELSEIF (TY(K).EQ.2) THEN
               PROD = PROD*(1-SURV(TY(K),(LMU(M)*ER(K))**(1./EBETA(K)),
     +                        EBETA(K),UT(K)))
               ELSEIF (TY(K).EQ.3) THEN
                 PROD=PROD*(1-SURV(TY(K),1.D0-LMU(M)*ER(K),0.D0,UT(K)))
               ENDIF
  115       CONTINUE
            REL2(L) = 1.0 - PROD
            RSS = RSS*REL2(L)
  120    CONTINUE
```

```
         L = L - 1
         RP = 1.0 - (1.0 - RSS)*(1.0 - RP)
         RSS =1.0
         IF (L.GE.(C + CSSGROUP(L))) THEN
            RPP = RPP * RP
            RP = 0.0
            C = C + CSSGROUP(L)
         ENDIF
         IF (L.GE.(CC + CSGROUP(L))) THEN
            RSP = 1.0 - (1.0 - RPP) * (1.0 - RSP )
            RPP = 1.0
            CC = CC + CSGROUP(L)
         ENDIF
         IF (L.GE.(CCC + NCGROUP(L))) THEN
            RSL(M) = RSL(M) * RSP
            RSP = 0.0
            CCC = CCC + NCGROUP(L)
         ENDIF
         J = J + CSERIES(L)
         IF (L.GE.NCOMP) TEMP = .TRUE.
      ENDDO
c
C  Increment replication counter.
c
      M = M + 1
c
      ENDDO
c
      RETURN
      END
c-----------------------------------------------------------------------
c 4. Subroutine for Evaluation.
c-----------------------------------------------------------------------
      SUBROUTINE EVAL
c
c  This subroutine calls BUBBLE to sort the array RSL(NREP) in
c  ascending order to get an ordered array ORSL(NREP). It also
c  determine the estimate for RSLOW at the specified significance
c  level ALPHA and the value of Level in which ORSL(LEVEL) is closest
c  to the true reliability RS.
c
c  Include the declaration files
c  and declare the local variables.
c
      INCLUDE 'PARM DEF'
c
      INTEGER  INDEX, M
      REAL*8 DIFF
c
c  Order the array RSL(NREP) in ascending order.
c
      DO 10 M = 1, NREP
         ORSL(M) = RSL(M)
   10 CONTINUE
c
```

```
c  Bubble Sort. Sink the larger of the pair.
c
      CALL BUBBLE(NREP,RSL,ORSL)
c
c  Determine the (1-ALPHA) % lower confidence bound for the system
c  reliability.
c
      RSLOW = ORSL(NINT(NREP*(1-ALPHA)))
c
c  Finding the % confidence level for the true reliability RS.
c  (i.e the proportion of RSL(M) lesser than RS)
c
      DIFF = 1.0
      INDEX = 0
      DO 200 M = 1, NREP
         IF (ABS(ORSL(M) - RS).LT.DIFF) THEN
            DIFF = ABS(ORSL(M) - RS)
            INDEX = M
         ENDIF
 200  CONTINUE
c
      LEVEL = FLOAT(INDEX)/NREP
c
c  Record evaluated parameters in RAW1.DAT (unit 2).
c
      OPEN(UNIT=2)
      WRITE(2,300)
 300  FORMAT(1x, '    M      LMU(M)    ELMAX(M)      RSL(M)',
     +          '    ORSL(M)    NFC(M)')
      DO 500 M = 1, NREP
         WRITE(2,400) M,LMU(M),ELMAX(M),RSL(M),ORSL(M),NFC(M)
 400     FORMAT(1x,I6,2F12.7,2F12.7,I6)
 500  CONTINUE
      CLOSE(UNIT=2)
c
      RETURN
      END
c------------------------------------------------------------------------
c 5. Subroutine for Report Generation.
c------------------------------------------------------------------------
      SUBROUTINE REPORT
c
c  This subroutine record the simulation result into the 'OUT DATA'
c  file indicated by logic 3.
c
c  Include the declaration files
c  and declare local variables.
c
      INCLUDE ' PARM DEF'
      INTEGER I, J, K, DUM(10)
c
c  Write to output file 'OUT DATA' designated as logic unit 3.
c
      OPEN(UNIT=3)
c
      WRITE(3,10)
```

```
      WRITE(3,20) NREP
      WRITE(3,25) NCOMP,CONFIG
      IF (CONFIG.EQ.1) THEN
         WRITE(3,26)
      ELSEIF (CONFIG.EQ.2) THEN
         WRITE(3,27)
      ELSE
         WRITE(3,28)
      ENDIF
      WRITE(3,29) DFR
      WRITE(3,30)
      WRITE(3,40)
      WRITE(3,50) ISEED,NCOMP,ALPHA,NCS,TCN
c
      WRITE(3,60)
      DO 200 I = 1, NCOMP
         WRITE(3,70) I,TY(I),PARM(1,I),PARM(2,I),UT(I),NC(I),NF(I)
 200  CONTINUE
      WRITE(3,80)
      WRITE(3,90)
      DO 300 I = 1, NCOMP
         WRITE(3,100) I,NF(I),TT(I),ELM(I),ER(I),EBETA(I)
 300  CONTINUE
      WRITE(3,110)
      WRITE(3,120)
      DO 500 J = 1, NCS
         DO 400 K = 1, 10
            DUM(K) = COMP(J,K)
 400     CONTINUE
      WRITE(3,130) J,DUM,REL1(J),REL2(J)
 500  CONTINUE
      WRITE(3,140)
      WRITE(3,150) RS,ELMAX(NREP),LMU(NREP),RSLOW,LEVEL
c
  10  FORMAT(1x,'OUT1 DATA : Output File of the RETP1 simulation')
  20  FORMAT(1x,'            after ',I5,' replication',/)
  25  FORMAT(1x,'COMMENTS  : ',I2,' COMPONENTS IN CONFIGURATION: ',I2 )
  26  FORMAT(1x,'            (SERIES SYSTEM)    ',/)
  27  FORMAT(1x,'            (PARALLEL SYSTEM)    ',/)
  28  FORMAT(1x,'            (SERIES-PARALLEL SYSTEM)    ',/)
  29  FORMAT(1x,'            DF = 2 * (NFC - NCOMP) =',I4 )
  30  FORMAT(1x,'Input parameter: ',/)
  40  FORMAT(1x,'     ISEED   NCOMP   ALPHA    NCS   TCN',/)
  50  FORMAT(1x,F10.1,I8,F8.4,2I6,/)
  60  FORMAT(1x,' I   TY(I) PARM1(I) PARM2(I)  UT(I)    NC(I)  NF(I)',/)
  70  FORMAT(1x,I2,I6,1x,2F9.5,F8.2,2x,2I6)
  80  FORMAT(1x,/,'Output Parameters for the Last Replication: ',/)
  90  FORMAT(1x,'  I  NF(I)       TT(I)      ELM(I)      ER(I) ',
     +          '   EBETA(I)',/)
 100  FORMAT(1x,I2,I6,4x,E14.7,2x,F9.7,2x,F9.7,2x,F9.7)
 110  FORMAT(1x,/,'Cut-Set Data: ',/)
 120  FORMAT(1x,'  J  NUM      Component List      ',
     +          '   REL1(J)     REL2(M)   ',/)
 130  FORMAT(1x,I2,I5,9I3,2F12.9)
 140  FORMAT(1x,/,'        RS      ELMAX(M)     LMU(M)  ',
     +          '      RSLOW       LEVEL   ',/)
```

```
 150  FORMAT(1x,5F12.7,/)
c
      CLOSE(UNIT = 3)
c
      RETURN
      END
c------------------------------------------------------------------------
c  This portion of the file contains functions and subroutines
c  used in the RETP model.
c-------------------------------------------------------------------------
c A. Random Number Generating Subroutine (LRNDPC).
c-------------------------------------------------------------------------
      SUBROUTINE LRNDPC (DSEED,U,N)
c
      INTEGER  N, I
      REAL*8   U(N)
      REAL*8   D31M1, DSEED, D31
c
c
      DATA D31M1/2147483647.D0/
      DATA D31  /2147483648.D0/
      DO 5 I = 1, N
         DSEED = DMOD(16807.D0*DSEED,D31M1)
  5   U(I) = DSEED / D31
      RETURN
      END
c-------------------------------------------------------------------------
c B. Survivability Function.
c-------------------------------------------------------------------------
      FUNCTION SURV(TYPE,PAR1,PAR2,UTIL)
c
c  This function returns the survival probability of the component of
c  different types (TYPE) with scale (PAR1) and shape (PAR2) parameters
c  given the specified utilization times or cycles (UTIL).
c
      INTEGER  TYPE, N
      REAL*8   PAR1, PAR2, UTIL
c
      IF (TYPE.EQ.1) THEN
         SURV = EXP(-(PAR1*UTIL))
      ELSEIF (TYPE.EQ.2) THEN
         SURV = EXP(-((PAR1*UTIL)**PAR2))
      ELSE
         N = NINT(UTIL)
         SURV = PAR1**N
      ENDIF
      END
c-------------------------------------------------------------------------
c C.  Bubble Sort Routine in Ascending Order.
c-------------------------------------------------------------------------
      SUBROUTINE BUBBLE(N,LIST,OLIST)
c
c  This subroutine performs a bubble sort in increasing order.
c  (i.e sink the greater numeral) for the first N terms in an array
c  LIST and returns the result in OLIST.
c
```

```
      LOGICAL DONE
      INTEGER N, K, PAIR
      REAL    TEMP
      REAL*8  LIST(*), OLIST(*)
c
c
c  Sink the larger of the pair.
c
      DO 50 K = 1, N
         OLIST(K) = LIST(K)
  50  CONTINUE
      PAIR = N - 1
      DONE = .FALSE.
      DO WHILE (.NOT.DONE)
         DONE = .TRUE.
         DO 100 K = 1, PAIR
            IF (OLIST(K).GT.OLIST(K+1)) THEN
               TEMP = OLIST(K)
               OLIST(K) = OLIST(K+1)
               OLIST(K+1) = TEMP
               DONE = .FALSE.
             ENDIF
 100     CONTINUE
         PAIR = PAIR - 1
      ENDDO
      RETURN
      END
c-------------------------------------------------------------------------
c D. Unbiasing Factor for Biased MLE for Weibull Shape Parameter.
c-------------------------------------------------------------------------
      FUNCTION BN(I)
c
c  This function returns the value of the unbiased factor for the biased
c  approximate MLE of the shape parameter of a Weibull distribution
c  with a sample size of N.
c
c
      INTEGER I
c
      IF (I.LE.5) THEN
         BN = (I*0.699)/(5.0)
      ELSEIF (I.EQ.6) THEN
         BN = 0.752
      ELSEIF (I.EQ.7) THEN
         BN = 0.786
      ELSEIF (I.EQ.8) THEN
         BN = 0.82
      ELSEIF (I.EQ.9) THEN
         BN = 0.8395

      ELSEIF (I.EQ.10) THEN
         BN = 0.859
      ELSEIF (I.EQ.11) THEN
         BN = 0.871
      ELSEIF (I.EQ.12) THEN
         BN = 0.883
```

```
      ELSEIF (I.EQ.13) THEN
         BN = 0.892
      ELSEIF (I.EQ.14) THEN
         BN = 0.901
      ELSEIF (I.EQ.15) THEN
         BN = 0.9075
      ELSEIF (I.EQ.16) THEN
         BN = 0.914
      ELSEIF (I.EQ.17) THEN
         BN = 0.9185
      ELSEIF (I.EQ.18) THEN
         BN = 0.923
      ELSEIF (I.EQ.19) THEN
         BN = 0.927
      ELSEIF (I.EQ.20) THEN
         BN = 0.931
      ELSEIF (I.LE.25) THEN
         BN = 0.931 + (I-20)*0.014/5.0
      ELSEIF (I.LE.30) THEN
         BN = 0.945 + (I-25)*0.01/5.0
      ELSEIF (I.LE.40) THEN
         BN = 0.955 + (I-30)*0.011/10.0
      ELSEIF (I.LE.60) THEN
         BN = 0.966 + (I-40)*0.012/20.0
      ELSEIF (I.LE.80) THEN
         BN = 0.978 + (I-60)*0.006/20.0
      ELSEIF (I.LE.100) THEN
         BN = 0.984 + (I-80)*0.003/20.0
      ELSEIF (I.LE.120) THEN
         BN = 0.987 + (I-100)*0.003/20.0
      ELSE
        BN = 1.0
      ENDIF
      RETURN
      END
c-------------------------------------------------------------------------
c E.Unbiasing Factor for approx MLE for scale parameter.
c-------------------------------------------------------------------------
      FUNCTION BIAS(N,S)
c
c  This function returns the value of the unbiased factor for the biased
c  approximate MLE of the scale parameter of a extreme value distribution
c  with a sample size of N.
c
c
      INTEGER N, S
c
      IF (N.EQ.10) THEN
         IF (S.EQ.0) THEN
            BIAS = 0.9339
         ELSEIF (S.EQ.1) THEN
            BIAS = 0.9275
         ELSEIF (S.EQ.2) THEN
            BIAS = 0.9152
         ELSEIF (S.EQ.3) THEN
            BIAS = 0.9001
```

```
         ELSEIF (S.EQ.4) THEN
            BIAS = 0.8908
         ELSEIF (S.EQ.8) THEN
            BIAS = 0.8453
         ELSE
            BIAS = 0.7998
         ENDIF
      ELSEIF (N.EQ.15) THEN
         IF (S.EQ.0) THEN
            BIAS = 0.95025
         ELSEIF (S.EQ.1) THEN
            BIAS = 0.94715
         ELSEIF (S.EQ.2) THEN
            BIAS = 0.94015
         ELSEIF (S.EQ.3) THEN
            BIAS = 0.9309
         ELSEIF (S.EQ.4) THEN
            BIAS = 0.92495
         ELSEIF (S.EQ.8) THEN
            BIAS = 0.89855
         ELSE
            BIAS = 0.8718
         ENDIF
      ELSEIF (N.EQ.20) THEN
         IF (S.EQ.0) THEN
            BIAS = 0.9666
         ELSEIF (S.EQ.1) THEN
            BIAS = 0.9668
         ELSEIF (S.EQ.2) THEN
            BIAS = 0.9651
         ELSEIF (S.EQ.3) THEN
            BIAS = 0.9617
         ELSEIF (S.EQ.4) THEN
            BIAS = 0.9591
         ELSEIF (S.EQ.8) THEN
            BIAS = 0.9518
         ELSE
            BIAS = 0.9438
         ENDIF
      ENDIF
      RETURN
      END
c-----------------------------------------------------------------------
c F. Biased APROXIMATE MLE Weibull Shape Parameter.
c-----------------------------------------------------------------------
      SUBROUTINE APROXMLE(Y,NN,A,BETAHAT)
c
      INTEGER I, NN, R, S, A
      REAL*8 Y(*), P(20), Q(20), ALPA(20), BETA(20),
     +       BETAHAT,GAMMA,DELTA,B,C,D,E,M,BB,CC,DD,EE,MM,SIGMAHAT
c

      R = 0
      S = NN - R - A
c
      DO 5 I=1, A
```

```
         P(I) = REAL(I)/(NN + 1)
         Q(I) = 1.0 - P(I)
         ALPA(I)=1.0 + LOG(Q(I))*(1.0 - LOG(-LOG(Q(I))))
         BETA(I)= -LOG(Q(I))
  5   CONTINUE
c
      GAMMA = -(Q(R+1)/P(R+1))*LOG(Q(R+1))*(1.0-LOG(-LOG(Q(R+1)))) +
     +         (Q(R+1)/(P(R+1)**2))*((LOG(Q(R+1)))**2)*LOG(-LOG(Q(R+1)))
c
      DELTA = (Q(R+1)/P(R+1))*LOG(Q(R+1))*(1.0+(LOG(Q(R+1))/P(R+1)))
c
      BB = 0.0
      CC = 0.0
      DD = 0.0
      EE = 0.0
      MM = 0.0
      DO 10 I=R+1, NN-S
         CC = CC + ALPA(I)
         BB = BB + BETA(I)*Y(I)
         MM = MM + BETA(I)
 10   CONTINUE
c
      M = R*DELTA + S*BETA(NN-S) + MM
      B = (R*DELTA*Y(R+1) + S*BETA(NN-S)*Y(NN-S) + BB)/M
      C = (R*GAMMA - S*(1.0 - ALPA(NN-S)) + CC)/M
c
      DO 15 I=R+1, NN-S
         DD = DD + ALPA(I)*(Y(I) - B)
         EE = EE + BETA(I)*((Y(I) - B)**2)
 15   CONTINUE
c
      D = R*GAMMA*(Y(R+1)-B) - S*(1.0-ALPA(NN-S))*(Y(NN-S)-B) + DD
      E = R*DELTA*((Y(R+1)-B)**2) + S*BETA(NN-S)*((Y(NN-S)-B)**2) + EE
c
      SIGMAHAT =(-D + SQRT(D**2 + 4.0*E*FLOAT(A)))/(2.0*FLOAT(A))
      SIGMAHAT =SIGMAHAT/BIAS(NN,S)
      BETAHAT = 1.0/SIGMAHAT
c
      RETURN
      END
c-----------------------------------------------------------------------
c G.  Chi-Square Quantile Function.
c-----------------------------------------------------------------------
      FUNCTION CHISQD(P,N)
c
c  Modified version of Algorithm 451 from Communication of the ACM
c  August 1977 Vol.16 No.8.
c
c  This function evaluates the quantile at the probability level P
c  (left tail area) for the Chi-Square Distribution with
c  N degrees of freedom.
c
c
      REAL*8 P
      REAL  X
      INTEGER  IF
```

```
      DIMENSION C(21), A(19)
c
      DATA C/ 1.565326E-3,
     +        1.060438E-3,
     +       -6.950356E-3,
     +       -1.323293E-2,
     +        2.277679E-2,
     +       -8.986007E-3,
     +       -1.513904E-2,
     +        2.530010E-3,
     +       -1.450117E-3,
     +        5.169654E-3,
     +       -1.153761E-2,
     +        1.128186E-2,
     +        2.607083E-2,
     +       -0.2237368,
     +        9.780499E-5,
     +       -8.426812E-4,
     +        3.125580E-3,
     +       -8.553069E-3,
     +        1.348028E-4,
     +        0.4713941,
     +        1.0000886 /
c
      DATA A/ 1.264616E-2,
     +       -1.425296E-2,
     +        1.400483E-2,
     +       -5.886090E-3,
     +       -1.091214E-2,
     +       -2.304527E-2,
     +        3.135411E-3,
     +       -2.728484E-4,
     +       -9.699681E-3,
     +        1.316872E-2,
     +        2.618914E-2,
     +       -0.2222222,
     +        5.406674E-5,
     +        3.483789E-5,
     +       -7.274761E-4,
     +        3.292181E-3,
     +       -8.729713E-3,
     +        0.4714045,
     +        1./
c
      IF (N-2) 10, 20, 30
   10 CALL XFROMP(.5*(1.-P),X,IF)
      CHISQD = X
      CHISQD = CHISQD*CHISQD
      RETURN
c
   20 CHISQD = -2.*LOG(1.-P)
      RETURN
c
   30 F = N
      F1 = 1./F
      CALL XFROMP(P,X,IF)
```

```
      T = X
      F2 = SQRT(F1)*T
      IF (N.GE.(2+INT(4.*ABS(T)))) GO TO 40
c
      CHISQD = (((((((C(1)*F2+C(2))*F2+C(3))*F2+C(4))*F2
     +          +C(5))*F2+C(6))*F2+C(7))*F1+((((((C(8)+C(9)*F2)*F2
     +          +C(10))*F2+C(11))*F2+C(12))*F2+C(13))*F2+C(14)))*F1+
     +          (((((C(15)*F2+C(16))*F2+C(17))*F2+C(18))*F2
     +          +C(19))*F2+C(20))*F2+C(21)
c
      GO TO 50
c
   40 CHISQD = (((A(1)+A(2)*F2)*F1+(((A(3)+A(4)*F2)*F2
     +          +A(5))*F2+A(6)))*F1+(((((A(7)+A(8)*F2)*F2+A(9))*F2
     +          +A(10))*F2+A(11))*F2+A(12)))*F1+(((((A(13)*F2
     +          +A(14))*F2+A(15))*F2+A(16))*F2+A(17))*F2*F2
     +          +A(18))*F2+A(19)
c
   50 CHISQD = CHISQD*CHISQD*CHISQD*F
C
      RETURN
      END
c-----------------------------------------------------------------------
c H. Standard Normal Variate Computation Subroutine.
c-----------------------------------------------------------------------
      SUBROUTINE XFROMP(P,X,IFAULT)
c
c  Algorithm AS 24 J.R.STAT.SOC. C (1969) Vol.18. No.3.
c
c  This subroutine compute the standard normal deviate X for the
c  specified left tail area P.
c
c
      REAL*8 P
      DIMENSION A(5)
      DIMENSION CONNOR (17), HSTNGS(6)
c
      DATA CONNOR/ 8.0327350124E-17,
     +             1.4483264644E-15,
     +             2.4668270103E-14,
     +             3.9554295164E-13,
     +             5.9477940136E-12,
     +             8.3507027951E-11,
     +             1.0892221037E-9,
     +             1.3122532964E-8,
     +             1.4503852223E-7,
     +             1.4589169001E-6,
     +             1.3227513228E-5,
     +             1.0683760684E-4,
     +             7.5757575758E-4,
     +             4.6296296296E-3,
     +             2.3809523810E-2,
     +             0.1,
     +             0.33333333333 /
c
```

```
      DATA RTHFPI / 1.2533141373 /
c
      DATA RRT2PI / 0.3989422804 /
c
      DATA TERMIN / 1.0E-11 /
c
      DATA HSTNGS / 2.515517,
     +              0.802853,
     +              0.010328,
     +              1.432788,
     +              0.189269,
     +              0.001308 /
c
      IFAULT = 1
      IF ((P.LE.0.0).OR.(P.GE.1.0)) GO TO 100
      IFAULT = 0
c
c  Get first approximation Xo to deviate by Hasting's formula.
c
      B = P
      IF (B.GT.0.5) B = 1.0 - B
c
      F = - LOG(B)
      E = SQRT(F+F)
c
      XO = -E + ((HSTNGS(3)*E+HSTNGS(2))*E+HSTNGS(1))/
     +     (((HSTNGS(6)*E+HSTNGS(5))*E+HSTNGS(4))*E+1.0)
c
      IF (XO.LT.0.0) GO TO 1
      XO = 0.0
      PO = 0.5
      X1 = -RTHFPI
      GO TO 7
c
c  Find the area PO corresponding to XO
c
  1   Y = XO**2
      IF (XO.LE.-1.9) GO TO 3
      Y = -0.5*Y
c
c  (1) series approximation
c
      PO = CONNOR(1)
      DO 2 L = 2, 17
  2   PO = PO*Y + CONNOR(L)
      PO = (PO*Y+1.0)*XO
      X1 = -(PO+RTHFPI)*EXP(-Y)
      PO = PO*RRT2PI + 0.5
      GO TO 7
c
c  (2) continued fraction approximation
c
  3   Z = 1.0/Y
      A(2) = 1.0
      A(3) = 1.0
      A(4) = Z + 1.0
```

```
      A(5) = 1.0
      W = 2.0
c
  4   DO 6 L = 1, 3, 2
         DO 5 J = 1, 2
            K = L + J
            KA= 7 - K
  5      A(K) = A(KA) + A(K)*W*Z
  6   W = W + 1.0
c
      APPRXU = A(2)/A(3)
      APPRXL = A(5)/A(4)
      C = APPRXU - APPRXL
c
      IF (C.GE.TERMIN) GO TO 4
c
      X1 = APPRXL/XO
      PO = -X1*RRT2PI*EXP(-0.5*Y)
c
c Get accurate value of deviate by Taylor Series.
c (X1, X2, X3 are derivatives for the Taylor series)
c
  7   D = F + LOG(PO)
      X2 = XO*X1*X1 - X1
      X3 = X1**3 + 2.0*XO*X1*X2 - X2
      X = ((X3*D/3.0+X2)*D/2.0+X1)*D + XO
c
      IF (P.LE.0.5) GO TO 100
      X = -X
 100  RETURN
      END
```

APPENDIX D. TABULATED RUN RESULTS FOR RETP

Table 1A : 8 Exp in Series, RS = 0.9305 (Hi)
min λ = 0.0002 f/hr, max λ = 0.0016 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	2NFC (160)	0.1	0.9262	0.9630
			0.2	0.9256	0.9220
		2(NFC+ NCOMP) (176)	0.1	0.9196	0.9970
			0.2	0.9188	0.9910
		2NFC- NCOMP (152)	0.1	0.9296	0.9209
			0.2	0.9291	0.8430
		2(NFC- NCOMP) (144)	0.1	0.9329	0.8400
			0.2	0.9325	0.7300
2	Test 15 until 15 failed NFC=120	2NFC (240)	0.1	0.9277	0.9550
			0.2	0.9273	0.9080
		2(NFC+ NCOMP) (256)	0.1	0.9233	0.9899
			0.2	0.9228	0.9750
		2NFC- NCOMP (232)	0.1	0.9298	0.9159
			0.2	0.9296	0.8329
		2(NFC- NCOMP) (224)	0.1	0.9321	0.8440
			0.2	0.9318	0.7470
3	Test 15 until 11 failed NFC=88	2NFC (176)	0.1	0.9268	0.9550
			0.2	0.9262	0.9159
		2(NFC+ NCOMP) (192)	0.1	0.9208	0.9960
			0.2	0.9200	0.9880
		2NFC- NCOMP (168)	0.1	0.9298	0.9159
			0.2	0.9293	0.8430
		2(NFC- NCOMP) (160)	0.1	0.9328	0.8430
			0.2	0.9324	0.7350

Table 1A : 8 Exp in Series, RS = 0.9305 (Hi) (Cont...)
min λ = 0.0002 f/hr, max λ = 0.0016 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
4	Test 15 until 7 failed NFC=56	2NFC (112)	0.1	0.9241	0.9700
			0.2	0.9229	0.9310
		2(NFC+ NCOMP) (128)	0.1	0.9145	0.9980
			0.2	0.9130	0.9940
		2NFC- NCOMP (104)	0.1	0.9289	0.9190
			0.2	0.9280	0.8530
		2(NFC- NCOMP) (96)	0.1	0.9338	0.8350
			0.2	0.9331	0.7200
5	Test 15 until 3 failed NFC=24	2NFC (48)	0.1	0.9145	0.9859
			0.2	0.9129	0.9750
		2(NFC+ NCOMP) (64)	0.1	0.8907	1.0000
			0.2	0.8877	1.0000
		2NFC- NCOMP (40)	0.1	0.9268	0.9439
			0.2	0.9260	0.8600
		2(NFC- NCOMP) (32)	0.1	0.9394	0.7530
			0.2	0.9394	0.6339

Table 1B : 8 Exp in Series, RS = 0.8025 (Low)
min λ = 0.0010 f/hr, max λ = 0.0045 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	2NFC (160)	0.1	0.7907	0.9710
			0.2	0.7884	0.9310
		2(NFC+ NCOMP) (176)	0.1	0.7735	0.9990
			0.2	0.7707	0.9920
		2NFC- NCOMP (152)	0.1	0.7994	0.9299
			0.2	0.7975	0.8500
		2(NFC- NCOMP) (144)	0.1	0.8083	0.8480
			0.2	0.8066	0.7410
2	Test 15 until 15 failed NFC=120	2NFC (240)	0.1	0.7940	0.9620
			0.2	0.7933	0.9159
		2(NFC+ NCOMP) (256)	0.1	0.7826	0.9930
			0.2	0.7816	0.9809
		2NFC- NCOMP (232)	0.1	0.7998	0.9240
			0.2	0.7992	0.8400
		2(NFC- NCOMP) (224)	0.1	0.8057	0.8580
			0.2	0.8052	0.7550
3	Test 15 until 11 failed NFC=88	2NFC (176)	0.1	0.7919	0.9660
			0.2	0.7896	0.9209
		2(NFC+ NCOMP) (192)	0.1	0.7764	0.9960
			0.2	0.7735	0.9890
		2NFC- NCOMP (168)	0.1	0.7999	0.9220
			0.2	0.7978	0.8550
		2(NFC- NCOMP) (160)	0.1	0.8079	0.8550
			0.2	0.8061	0.7460

Table 1B : 8 Exp in Series, RS = 0.8025 (Low) (Cont...)
min λ = 0.0002 f/hr, max λ = 0.0016 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
4	Test 15 until 7 failed NFC = 56	2NFC (112)	0.1	0.7851	0.9740
			0.2	0.7816	0.9380
		2(NFC + NCOMP) (128)	0.1	0.7605	0.9980
			0.2	0.7559	0.9960
		2NFC-NCOMP (104)	0.1	0.7978	0.9250
			0.2	0.7948	0.8600
		2(NFC-NCOMP) (96)	0.1	0.8107	0.8410
			0.2	0.8083	0.7270
5	Test 15 until 3 failed NFC = 24	2NFC (48)	0.1	0.7593	0.9890
			0.2	0.7555	0.9770
		2(NFC + NCOMP) (64)	0.1	0.7001	1.0000
			0.2	0.6928	1.0000
		2NFC-NCOMP (40)	0.1	0.7912	0.9489
			0.2	0.7892	0.8720
		2(NFC-NCOMP) (32)	0.1	0.8249	0.7630
			0.2	0.8248	0.6420

Table 2A : 8 Wei in Series, RS = 0.9798 (Hi)
min λ = 0.001 f/hr, max λ = 0.008 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	2NFC (160)	0.1	0.9602	0.9859
			0.2	0.9506	0.9820
		2(NFC+ NCOMP) (176)	0.1	0.9565	0.9890
			0.2	0.9461	0.9870
		2NFC- NCOMP (152)	0.1	0.9619	0.9820
			0.2	0.9529	0.9809
		2(NFC- NCOMP) (144)	0.1	0.9638	0.9800
			0.2	0.9553	0.9770
2	Test 15 until 15 failed NFC=120	2NFC (240)	0.1	0.9706	0.9770
			0.2	0.9649	0.9719
		2(NFC+ NCOMP) (256)	0.1	0.9687	0.9820
			0.2	0.9627	0.9790
		2NFC- NCOMP (232)	0.1	0.9715	0.9730
			0.2	0.9659	0.9640
		2(NFC- NCOMP) (224)	0.1	0.9724	0.9650
			0.2	0.9671	0.9590
3	Test 15 until 11 failed NFC=88	2NFC (176)	0.1	0.9704	0.9719
			0.2	0.9637	0.9650
		2(NFC+ NCOMP) (192)	0.1	0.9679	0.9800
			0.2	0.9606	0.9760
		2NFC- NCOMP (168)	0.1	0.9716	0.9660
			0.2	0.9652	0.9590
		2(NFC- NCOMP) (160)	0.1	0.9729	0.9590
			0.2	0.9668	0.9510

Table 2A : 8 Wei in Series, RS = 0.9798 (Hi) (Cont...)
min λ = 0.001 f/hr, max λ = 0.008 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
4	Test 15 until 7 failed NFC=56	2NFC (112)	0.1	0.9764	0.9400
			0.2	0.9668	0.9280
		2(NFC+ NCOMP) (128)	0.1	0.9733	0.9579
			0.2	0.9624	0.9510
		2NFC- NCOMP (104)	0.1	0.9779	0.9240
			0.2	0.9691	0.9110
		2(NFC- NCOMP) (96)	0.1	0.9795	0.9080
			0.2	0.9713	0.8900
5	Test 15 until 3 failed NFC=24	2NFC (48)	0.1	0.9889	0.8000
			0.2	0.9814	0.7770
		2(NFC+ NCOMP) (64)	0.1	0.9857	0.8469
			0.2	0.9758	0.8360
		2NFC- NCOMP (40)	0.1	0.9906	0.7510
			0.2	0.9843	0.7280
		2(NFC- NCOMP) (32)	0.1	0.9922	0.6890
			0.2	0.9872	0.6530

Table 2B : 8 Wei in Series, RS = 0.8323 (Low)
min λ = 0.003 f/hr, max λ = 0.024 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	2NFC (160)	0.1	0.7616	0.9800
			0.2	0.7383	0.9740
		2(NFC+ NCOMP) (176)	0.1	0.7425	0.9880
			0.2	0.7171	0.9840
		2NFC- NCOMP (152)	0.1	0.7714	0.9730
			0.2	0.7491	0.9680
		2(NFC- NCOMP) (144)	0.1	0.7813	0.9680
			0.2	0.7601	0.9620
2	Test 15 until 15 failed NFC=120	2NFC (240)	0.1	0.8002	0.9640
			0.2	0.7766	0.9529
		2(NFC+ NCOMP) (256)	0.1	0.7891	0.9730
			0.2	0.7641	0.9669
		2NFC- NCOMP (232)	0.1	0.9059	0.9560
			0.2	0.7829	0.9430
		2(NFC- NCOMP) (224)	0.1	0.8116	0.9450
			0.2	0.7893	0.9320
3	Test 15 until 11 failed NFC=88	2NFC (176)	0.1	0.8056	0.9529
			0.2	0.7803	0.9430
		2(NFC+ NCOMP) (192)	0.1	0.7909	0.9710
			0.2	0.7636	0.9640
		2NFC- NCOMP (168)	0.1	0.8131	0.9430
			0.2	0.7888	0.9270
		2(NFC- NCOMP) (160)	0.1	0.8206	0.9270
			0.2	0.7994	0.9060

Table 2B : 8 Wei in Series, RS = 0.8323 (Low) (Cont...)
min λ = 0.003 f/hr, max λ = 0.024 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
4	Test 15 until 7 failed NFC = 56	2NFC (112)	0.1	0.8296	0.9030
			0.2	0.8064	0.8830
		2(NFC+ NCOMP) (128)	0.1	0.8095	0.9470
			0.2	0.7832	0.9320
		2NFC-NCOMP (104)	0.1	0.8399	0.8800
			0.2	0.8183	0.8540
		2(NFC-NCOMP) (96)	0.1	0.8504	0.8480
			0.2	0.8304	0.8070
5	Test 15 until 3 failed NFC = 24	2NFC (48)	0.1	0.8801	0.7449
			0.2	0.8569	0.7060
		2(NFC+ NCOMP) (64)	0.1	0.8475	0.8660
			0.2	0.8171	0.8410
		2NFC-NCOMP (40)	0.1	0.8970	0.6540
			0.2	0.8778	0.5969
		2(NFC-NCOMP) (32)	0.1	0.9145	0.5270
			0.2	0.8994	0.4470

Table 3A : 4 Exp and 4 Wei (mixed) in Series, RS = 0.9801 (Hi)
min λ = 0.0002 f/hr, max λ = 0.0008 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	2NFC (160)	0.1	0.9796	0.9190
			0.2	0.9791	0.8720
		2(NFC+ NCOMP) (176)	0.1	0.9777	0.9740
			0.2	0.9771	0.9489
		2NFC- NCOMP (152)	0.1	0.9805	0.8720
			0.2	0.9801	0.7980
		2(NFC- NCOMP) (144)	0.1	0.9815	0.7950
			0.2	0.9811	0.7000
2	Test 15 until 15 failed NFC=120	2NFC (240)	0.1	0.9802	0.8950
			0.2	0.9799	0.8250
		2(NFC+ NCOMP) (256)	0.1	0.9789	0.9510
			0.2	0.9786	0.9209
		2NFC- NCOMP (232)	0.1	0.9808	0.8370
			0.2	0.9805	0.7530
		2(NFC- NCOMP) (224)	0.1	0.9814	0.7670
			0.2	0.9812	0.6670
3	Test 15 until 11 failed NFC=88	2NFC (176)	0.1	0.9802	0.8920
			0.2	0.9797	0.8400
		2(NFC+ NCOMP) (192)	0.1	0.9785	0.9590
			0.2	0.9779	0.9320
		2NFC- NCOMP (168)	0.1	0.9810	0.8410
			0.2	0.9806	0.7550
		2(NFC- NCOMP) (160)	0.1	0.9819	0.7580
			0.2	0.9815	0.6590

Table 3A : 4 Exp and 4 Wei in Series, RS = 0.9801 (Hi) (Cont...)
min λ = 0.0002 f/hr, max λ = 0.0008 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
4	Test 15 until 7 failed NFC=56	2NFC (112)	0.1	0.9803	0.8950
			0.2	0.9797	0.8260
		2(NFC+ NCOMP) (128)	0.1	0.9777	0.9690
			0.2	0.9770	0.9439
		2NFC- NCOMP (104)	0.1	0.9815	0.8060
			0.2	0.9811	0.7300
		2(NFC- NCOMP) (96)	0.1	0.9828	0.7090
			0.2	0.9825	0.6000
5	Test 15 until 3 failed NFC=24	2NFC (48)	0.1	0.9815	0.8730
			0.2	0.9800	0.8020
		2(NFC+ NCOMP) (64)	0.1	0.9757	0.9740
			0.2	0.9739	0.9560
		2NFC- NCOMP (40)	0.1	0.9839	0.7250
			0.2	0.9831	0.6260
		2(NFC- NCOMP) (32)	0.1	0.9868	0.5150
			0.2	0.9862	0.4140

Table 3B : 4 Exp and 4 Wei (mixed) in Series, RS = 0.8089 (Low)
min λ = 0.002 f/hr, max λ = 0.008 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	2NFC (160)	0.1	0.7957	0.9570
			0.2	0.7914	0.9240
		2(NFC+ NCOMP) (176)	0.1	0.7789	0.9920
			0.2	0.7738	0.9770
		2NFC- NCOMP (152)	0.1	0.8042	0.9240
			0.2	0.8003	0.8730
		2(NFC- NCOMP) (144)	0.1	0.8129	0.8710
			0.2	0.8093	0.7950
2	Test 15 until 15 failed NFC=120	2NFC (240)	0.1	0.8033	0.9340
			0.2	0.8007	0.8929
		2(NFC+ NCOMP) (256)	0.1	0.7923	0.9740
			0.2	0.7893	0.9500
		2NFC- NCOMP (232)	0.1	0.8089	0.9010
			0.2	0.8064	0.8260
		2(NFC- NCOMP) (224)	0.1	0.8145	0.8400
			0.2	0.8122	0.7750
3	Test 15 until 11 failed NFC=88	2NFC (176)	0.1	0.8045	0.9290
			0.2	0.7993	0.8810
		2(NFC+ NCOMP) (192)	0.1	0.7897	0.9850
			0.2	0.7839	0.9570
		2NFC- NCOMP (168)	0.1	0.8119	0.8820
			0.2	0.8072	0.8189
		2(NFC- NCOMP) (160)	0.1	0.8195	0.8200
			0.2	0.8151	0.7300

Table 3B : 4 Exp and 4 Wei in Series, RS = 0.8089 (Low) (Cont...)
min λ = 0.002 f/hr, max λ = 0.008 f/hr, UT = 10 hrs.

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
4	Test 15 until 7 failed NFC=56	2NFC (112)	0.1	0.8056	0.9140
			0.2	0.8017	0.8600
		2(NFC+ NCOMP) (128)	0.1	0.7830	0.9760
			0.2	0.7780	0.9610
		2NFC- NCOMP (104)	0.1	0.8172	0.8460
			0.2	0.8139	0.7630
		2(NFC- NCOMP) (96)	0.1	0.8290	0.7410
			0.2	0.8262	0.6490
5	Test 15 until 3 failed NFC=24	2NFC (48)	0.1	0.8174	0.8670
			0.2	0.8085	0.8060
		2(NFC+ NCOMP) (64)	0.1	0.7703	0.9780
			0.2	0.7571	0.9550
		2NFC- NCOMP (40)	0.1	0.8424	0.7219
			0.2	0.8357	0.6050
		2(NFC- NCOMP) (32)	0.1	0.8685	0.4859
			0.2	0.8641	0.3810

Table 4A : 8 Exponential in Parallel, RS = 0.9345 (Hi)
min λ = 0.1000 f/hr, max λ = 0.1525 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	160	0.1	0.9306	0.9290
			0.2	0.9320	0.8180
2	Test 15 until 15 failed NFC=120	240	0.1	0.9325	0.9190
			0.2	0.9324	0.8210
3	Test 15 until 11 failed NFC=88	176	0.1	0.9309	0.9240
			0.2	0.9312	0.8329
4	Test 15 until 7 failed NFC=56	112	0.1	0.9307	0.9209
			0.2	0.9308	0.8310
5	Test 15 until 3 failed NFC=24	48	0.1	0.9280	0.9220
			0.2	0.9286	0.8370

Table 4B : 8 Exponential in Parallel, RS = 0.8262 (Low)
min λ = 0.1000 f/hr, max λ = 0.2575 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC = 80	160	0.1	0.8309	0.8870
			0.2	0.8288	0.7880
2	Test 15 until 15 failed NFC = 120	240	0.1	0.8290	0.8870
			0.2	0.8283	0.7860
3	Test 15 until 11 failed NFC = 88	176	0.1	0.8266	0.8990
			0.2	0.8264	0.7990
4	Test 15 until 7 failed NFC = 56	112	0.1	0.8300	0.8860
			0.2	0.8284	0.7890
5	Test 15 until 3 failed NFC = 24	48	0.1	0.8354	0.8810
			0.2	0.8364	0.7700

Table 5A : 8 Wei in Parallel , RS = 0.9265 (Hi)
min λ = 0.100 f/hr, max λ = 0.128 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	161	0.1	0.9154	0.9540
			0.2	0.9157	0.8880
2	Test 15 until 15 failed NFC=120	241	0.1	0.9179	0.9510
			0.2	0.9170	0.8770
3	Test 15 until 11 failed NFC=88	177	0.1	0.9248	0.9180
			0.2	0.9250	0.8130
4	Test 15 until 7 failed NFC=56	113	0.1	0.9323	0.8630
			0.2	0.9318	0.7589
5	Test 15 until 3 failed NFC=24	51	0.1	0.9152	0.9270
			0.2	0.8980	0.8850

Table 5B : 8 Wei in Parallel , RS = 0.8351 (Low)
min λ = 0.100 f/hr, max λ = 0.163 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	161	0.1	0.8296	0.9180
			0.2	0.8331	0.8140
2	Test 15 until 15 failed NFC=120	241	0.1	0.8322	0.9100
			0.2	0.8314	0.8279
3	Test 15 until 11 failed NFC=88	177	0.1	0.8494	0.8380
			0.2	0.8489	0.7130
4	Test 15 until 7 failed NFC=56	113	0.1	0.8713	0.7560
			0.2	0.8705	0.6450
5	Test 15 until 3 failed NFC=24	51	0.1	0.8678	0.8360
			0.2	0.8469	0.7770

Table 6A : 4 EXP and 4 Wei (mixed) in Parallel, RS = 0.9408 (Hi)
min λ = 0.100 f/hr, max λ = 0.130 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	161	0.1	0.9342	0.9420
			0.2	0.9366	0.8500
2	Test 15 until 15 failed NFC=120	241	0.1	0.9362	0.9330
			0.2	0.9363	0.8510
3	Test 15 until 11 failed NFC=88	177	0.1	0.9389	0.9240
			0.2	0.9378	0.8310
4	Test 15 until 7 failed NFC=56	113	0.1	0.9416	0.8920
			0.2	0.9427	0.7819
5	Test 15 until 3 failed NFC=24	51	0.1	0.9383	0.9080
			0.2	0.9325	0.8410

Table 6B : 4 EXP and 4 Wei (mixed) in Parallel, RS = 0.8170 (Low)
min λ = 0.100 f/hr, max λ = 0.220 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	161	0.1	0.8264	0.8550
			0.2	0.8273	0.7460
2	Test 15 until 15 failed NFC=120	241	0.1	0.8262	0.8630
			0.2	0.8236	0.7560
3	Test 15 until 11 failed NFC=88	177	0.1	0.8333	0.8290
			0.2	0.8334	0.7150
4	Test 15 until 7 failed NFC=56	113	0.1	0.8488	0.7660
			0.2	0.8469	0.6470
5	Test 15 until 3 failed NFC=24	51	0.1	0.8650	0.7819
			0.2	0.8563	0.6709

Table 7A : 8 Exp in Series- Parallel, RS = 0.9249 (Hi)
min λ = 0.0003 f/hr, max λ = 0.0024 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC = 80	160	0.1	0.9241	0.9159
			0.2	0.9226	0.8620
2	Test 15 until 15 failed NFC = 120	240	0.1	0.9253	0.8960
			0.2	0.9236	0.8360
3	Test 15 until 11 failed NFC = 88	176	0.1	0.9252	0.8950
			0.2	0.9226	0.8390
4	Test 15 until 7 failed NFC = 56	112	0.1	0.9222	0.9260
			0.2	0.9204	0.8789
5	Test 15 until 3 failed NFC = 24	48	0.1	0.9153	0.9620
			0.2	0.9123	0.9260

Table 7B : 8 Exp in Series- Parallel, RS = 0.8228 (Low)
min λ = 0.00075 f/hr, max λ = 0.0060 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC = 80	160	0.1	0.8210	0.9159
			0.2	0.8175	0.8620
2	Test 15 until 15 failed NFC = 120	240	0.1	0.8237	0.8960
			0.2	0.8199	0.8360
3	Test 15 until 11 failed NFC = 88	176	0.1	0.8234	0.8950
			0.2	0.8176	0.8390
4	Test 15 until 7 failed NFC = 56	112	0.1	0.8168	0.9260
			0.2	0.8127	0.8789
5	Test 15 until 3 failed NFC = 24	48	0.1	0.8015	0.9620
			0.2	0.7949	0.9260

Table 8A : 8 Wel in Series- Parallel, RS = 0.9328 (Hi)
min λ = 0.002 f/hr, max λ = 0.016 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	161	0.1	0.9106	0.9579
			0.2	0.8939	0.9510
2	Test 15 until 15 failed NFC=120	241	0.1	0.9247	0.9330
			0.2	0.9108	0.9230
3	Test 15 until 11 failed NFC=88	177	0.1	0.9266	0.9220
			0.2	0.9149	0.9119
4	Test 15 until 7 failed NFC=56	113	0.1	0.9414	0.8609
			0.2	0.9250	0.8419
5	Test 15 until 3 failed NFC=24	51	0.1	0.9699	0.7079
			0.2	0.9536	0.6740

Table 8B : 8 Wel in Series- Parallel, RS = 0.8321 (Low)
min λ = 0.00325 f/hr, max λ = 0.026 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	161	0.1	0.7963	0.9560
			0.2	0.7703	0.9470
2	Test 15 until 15 failed NFC=120	241	0.1	0.8204	0.9310
			0.2	0.7986	0.9200
3	Test 15 until 11 failed NFC=88	177	0.1	0.8250	0.9190
			0.2	0.8046	0.9000
4	Test 15 until 7 failed NFC=56	113	0.1	0.8485	0.8590
			0.2	0.8237	0.8329
5	Test 15 until 3 failed NFC=24	51	0.1	0.8999	0.7500
			0.2	0.8599	0.6950

Table 9A : 4 EXP and 4 Wei in Series- Parallel, RS = 0.9276 (Hi)
min λ = 0.005 f/hr, max λ = 0.020 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	161	0.1	0.9096	0.9400
			0.2	0.8904	0.9349
2	Test 15 until 15 failed NFC=120	241	0.1	0.9238	0.9170
			0.2	0.9079	0.9040
3	Test 15 until 11 failed NFC=88	177	0.1	0.9266	0.9050
			0.2	0.9109	0.8850
4	Test 15 until 7 failed NFC=56	113	0.1	0.9393	0.8550
			0.2	0.9203	0.8390
5	Test 15 until 3 failed NFC=24	51	0.1	0.9637	0.7430
			0.2	0.9451	0.7200

Table 9B : 4 EXP and 4 Wei in Series- Parallel, RS = 0.8248 (Low)
min λ = 0.008 f/hr, max λ = 0.032 f/hr, UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=80	161	0.1	0.7909	0.9420
			0.2	0.7627	0.9360
2	Test 15 until 15 failed NFC=120	241	0.1	0.8164	0.9150
			0.2	0.7915	0.9020
3	Test 15 until 11 failed NFC=88	177	0.1	0.8236	0.9010
			0.2	0.7964	0.8820
4	Test 15 until 7 failed NFC=56	113	0.1	0.8412	0.8660
			0.2	0.8119	0.8419
5	Test 15 until 3 failed NFC=24	51	0.1	0.8683	0.8170
			0.2	0.8289	0.7940

Table 10A :10 EXP and 5 Wei in Series- Parallel, RS = 0.9472 (Hi)
Exp : min λ = 0.025 f/hr, max λ = 0.075 f/hr.
Wei : min λ = 0.055 f/hr, max λ = 0.095 f/hr.
UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=150	301	0.1	0.9383	0.9710
			0.2	0.9369	0.9550
2	Test 20 until 20 failed NFC=300	601	0.1	0.9449	0.9370
			0.2	0.9432	0.8950
3	Test 20 until 15 failed NFC=225	451	0.1	0.9444	0.9349
			0.2	0.9430	0.8979
4	Test 20 until 10 failed NFC=150	301	0.1	0.9435	0.9290
			0.2	0.9415	0.9000
5	Test 20 until 5 failed NFC=75	152	0.1	0.9344	0.9809
			0.2	0.9310	0.9520

Table 10B :10 EXP and 5 Wei in Series- Parallel, RS = 0.8324 (Low)
Exp : min λ = 0.025 f/hr, max λ = 0.175 f/hr.
Wei : min λ = 0.055 f/hr, max λ = 0.125 f/hr.
UT = 10 hrs.
Degrees of Freedom = 2NFC + 0.5(NC/NF)

S/N	Test Plan	Deg. of Freedom	α	Measures of accuracy	
				RSLOW	LEVEL
1	Test 10 until 10 failed NFC=150	301	0.1	0.8103	0.9669
			0.2	0.8081	0.9430
2	Test 20 until 20 failed NFC=300	601	0.1	0.8247	0.9380
			0.2	0.8219	0.8990
3	Test 20 until 15 failed NFC=225	451	0.1	0.8236	0.9470
			0.2	0.8204	0.8990
4	Test 20 until 10 failed NFC=150	301	0.1	0.8232	0.9500
			0.2	0.8166	0.8950
5	Test 20 until 5 failed NFC=75	152	0.1	0.8008	0.9760
			0.2	0.7917	0.9489

APPENDIX E. TABLE OF CHI-SQUARE DISTRIBUTION

Table 21 in this appendix provides the eightieth and ninetieth percentile points for the chi-square distribution with degrees of freedom ranging from 1 to 402 in increments of one. The percentile points appear under the column headed CHI in Table 21.

The computational algorithm used to construct these percentile points is defined in the Chi-Square Quantile Function routine together with the Standard Normal Variate Computation routine on pages 79 - 83 in this thesis.

Values of these chi-square percentile points for degrees of freedom as large as 601 were needed in this thesis. The extensive set of chi-square percentile values in Table 21 is provided as a convenience for a reader who may want to apply this methodology to a particular system.

Table 21 : Chi-square distribution

Left tail area = 0.9						Left tail area = 0.8					
DF	CHI	DF	CHI	DF	CHI	DF	CHI	DF	CHI	DF	CHI
1	2.706	47	59.774	93	110.850	1	1.642	47	54.905	93	104.241
2	4.605	48	60.907	94	111.944	2	3.219	48	55.993	94	105.303
3	6.253	49	62.037	95	113.038	3	4.644	49	57.078	95	106.364
4	7.781	50	63.167	96	114.130	4	5.990	50	58.164	96	107.425
5	9.237	51	64.295	97	115.223	5	7.289	51	59.248	97	108.486
6	10.645	52	65.422	98	116.315	6	8.558	52	60.331	98	109.547
7	12.017	53	66.548	99	117.407	7	9.803	53	61.414	99	110.607
8	13.361	54	67.673	100	118.498	8	11.030	54	62.496	100	111.666
9	14.684	55	68.796	101	119.589	9	12.242	55	63.577	101	112.726
10	15.987	56	69.918	102	120.678	10	13.442	56	64.658	102	113.785
11	17.275	57	71.039	103	121.768	11	14.631	57	65.737	103	114.844
12	18.549	58	72.160	104	122.858	12	15.812	58	66.816	104	115.903
13	19.812	59	73.279	105	123.946	13	16.985	59	67.894	105	116.961
14	21.064	60	74.397	106	125.035	14	18.151	60	68.972	106	118.019
15	22.307	61	75.514	107	126.123	15	19.311	61	70.049	107	119.077
16	23.542	62	76.630	108	127.211	16	20.465	62	71.125	108	120.135
17	24.769	63	77.745	109	128.298	17	21.614	63	72.201	109	121.192
18	25.989	64	78.859	110	129.385	18	22.759	64	73.276	110	122.249
19	27.204	65	79.973	111	130.471	19	23.900	65	74.350	111	123.306
20	28.412	66	81.085	112	131.558	20	25.037	66	75.424	112	124.363
21	29.615	67	82.197	113	132.643	21	26.171	67	76.498	113	125.419
22	30.813	68	83.308	114	133.728	22	27.301	68	77.571	114	126.475
23	32.007	69	84.418	115	134.813	23	28.429	69	78.643	115	127.531
24	33.196	70	85.527	116	135.898	24	29.553	70	79.714	116	128.586
25	34.382	71	86.635	117	136.982	25	30.675	71	80.786	117	129.642
26	35.563	72	87.743	118	138.066	26	31.795	72	81.856	118	130.697
27	36.741	73	88.850	119	139.149	27	32.912	73	82.927	119	131.751
28	37.916	74	89.956	120	140.232	28	34.026	74	83.996	120	132.806
29	39.087	75	91.061	121	141.315	29	35.139	75	85.066	121	133.860
30	40.256	76	92.166	122	142.397	30	36.250	76	86.134	122	134.914
31	41.422	77	93.270	123	143.480	31	37.359	77	87.203	123	135.968
32	42.585	78	94.373	124	144.561	32	38.466	78	88.271	124	137.022
33	43.745	79	95.476	125	145.643	33	39.572	79	89.338	125	138.076
34	44.903	80	96.578	126	146.724	34	40.676	80	90.405	126	139.129
35	46.059	81	97.679	127	147.805	35	41.778	81	91.472	127	140.182
36	47.212	82	98.780	128	148.885	36	42.879	82	92.538	128	141.235
37	48.363	83	99.880	129	149.965	37	43.978	83	93.604	129	142.288
38	49.513	84	100.980	130	151.045	38	45.076	84	94.669	130	143.340
39	50.660	85	102.079	131	152.125	39	46.173	85	95.734	131	144.392
40	51.805	86	103.177	132	153.204	40	47.268	86	96.799	132	145.444
41	52.948	87	104.275	133	154.282	41	48.363	87	97.863	133	146.496
42	54.090	88	105.372	134	155.361	42	49.456	88	98.927	134	147.547
43	55.230	89	106.469	135	156.440	43	50.548	89	99.990	135	148.599
44	56.368	90	107.565	136	157.517	44	51.639	90	101.053	136	149.650
45	57.505	91	108.660	137	158.595	45	52.729	91	102.116	137	150.701
46	58.640	92	109.756	138	159.673	46	53.818	92	103.179	138	151.752

Table 21 : Chi-square distribution (Continued...)

Left tail area = 0.9						Left tail area = 0.8					
DF	CHI	DF	CHI	DF	CHI	DF	CHI	DF	CHI	DF	CHI
139	160.750	183	207.906	227	254.698	139	152.803	183	198.876	227	244.710
140	161.827	184	208.972	228	255.758	140	153.853	184	199.920	228	245.750
141	162.903	185	210.039	229	256.818	141	154.904	185	200.964	229	246.789
142	163.979	186	211.106	230	257.878	142	155.954	186	202.007	230	247.828
143	165.055	187	212.172	231	258.938	143	157.004	187	203.051	231	248.868
144	166.131	188	213.238	232	259.997	144	158.054	188	204.095	232	249.907
145	167.207	189	214.304	233	261.057	145	159.103	189	205.138	233	250.946
146	168.282	190	215.370	234	262.117	146	160.153	190	206.181	234	251.985
147	169.357	191	216.436	235	263.176	147	161.202	191	207.225	235	253.024
148	170.432	192	217.502	236	264.235	148	162.251	192	208.268	236	254.063
149	171.507	193	218.567	237	265.293	149	163.300	193	209.311	237	255.101
150	172.581	194	219.632	238	266.353	150	164.349	194	210.354	238	256.140
151	173.655	195	220.698	239	267.411	151	165.397	195	211.396	239	257.179
152	174.729	196	221.763	240	268.470	152	166.446	196	212.439	240	258.217
153	175.802	197	222.827	241	269.529	153	167.494	197	213.482	241	259.255
154	176.875	198	223.892	242	270.587	154	168.542	198	214.524	242	260.294
155	177.949	199	224.956	243	271.645	155	169.590	199	215.566	243	261.332
156	179.021	200	226.021	244	272.703	156	170.638	200	216.608	244	262.371
157	180.094	201	227.085	245	273.761	157	171.686	201	217.651	245	263.408
158	181.167	202	228.149	246	274.819	158	172.733	202	218.693	246	264.447
159	182.238	203	229.213	247	275.877	159	173.781	203	219.734	247	265.485
160	183.310	204	230.276	248	276.935	160	174.828	204	220.776	248	266.522
161	184.382	205	231.339	249	277.992	161	175.875	205	221.818	249	267.560
162	185.453	206	232.403	250	279.050	162	176.922	206	222.859	250	268.598
163	186.524	207	233.466	251	280.107	163	177.969	207	223.901	251	269.635
164	187.596	208	234.529	252	281.164	164	179.015	208	224.942	252	270.673
165	188.666	209	235.591	253	282.221	165	180.062	209	225.984	253	271.710
166	189.737	210	236.654	254	283.278	166	181.108	210	227.024	254	272.748
167	190.807	211	237.717	255	284.335	167	182.154	211	228.066	255	273.786
168	191.878	212	238.779	256	285.392	168	183.200	212	229.107	256	274.823
169	192.947	213	239.842	257	286.448	169	184.246	213	230.148	257	275.860
170	194.017	214	240.903	258	287.505	170	185.292	214	231.188	258	276.897
171	195.086	215	241.966	259	288.562	171	186.338	215	232.229	259	277.934
172	196.156	216	243.027	260	289.618	172	187.383	216	233.269	260	278.971
173	197.225	217	244.089	261	290.674	173	188.429	217	234.310	261	280.008
174	198.294	218	245.151	262	291.730	174	189.474	218	235.351	262	281.045
175	199.362	219	246.212	263	292.786	175	190.519	219	236.390	263	282.082
176	200.431	220	247.273	264	293.842	176	191.564	220	237.431	264	283.119
177	201.499	221	248.334	265	294.898	177	192.609	221	238.472	265	284.155
178	202.567	222	249.396	266	295.953	178	193.654	222	239.511	266	285.192
179	203.636	223	250.456	267	297.010	179	194.698	223	240.551	267	286.228
180	204.703	224	251.517	268	298.064	180	195.743	224	241.591	268	287.264
181	205.771	225	252.577	269	299.120	181	196.787	225	242.631	269	288.301
182	206.838	226	253.637	270	300.175	182	197.832	226	243.670	270	289.337

Table 21 : Chi-square distribution (Continued...)

Left tail area = 0.9						Left tail area = 0.8					
DF	CHI	DF	CHI	DF	CHI	DF	CHI	DF	CHI	DF	CHI
271	301.230	315	347.564	359	393.738	271	290.374	315	335.906	359	381.334
272	302.286	316	348.615	360	394.787	272	291.409	316	336.940	360	382.366
273	303.340	317	349.666	361	395.834	273	292.445	317	337.973	361	383.397
274	304.395	318	350.717	362	396.882	274	293.481	318	339.007	362	384.429
275	305.450	319	351.768	363	397.930	275	294.518	319	340.039	363	385.459
276	306.505	320	352.819	364	398.978	276	295.553	320	341.073	364	386.491
277	307.559	321	353.869	365	400.024	277	296.589	321	342.106	365	387.522
278	308.614	322	354.919	366	401.071	278	297.625	322	343.139	366	388.554
279	309.668	323	355.969	367	402.119	279	298.661	323	344.173	367	389.584
280	310.722	324	357.020	368	403.167	280	299.697	324	345.206	368	390.615
281	311.777	325	358.071	369	404.214	281	300.732	325	346.239	369	391.646
282	312.831	326	359.120	370	405.261	282	301.768	326	347.272	370	392.678
283	313.885	327	360.171	371	406.308	283	302.803	327	348.306	371	393.708
284	314.938	328	361.222	372	407.356	284	303.839	328	349.338	372	394.740
285	315.992	329	362.271	373	408.402	285	304.874	329	350.371	373	395.771
286	317.045	330	363.322	374	409.449	286	305.909	330	351.403	374	396.801
287	318.100	331	364.371	375	410.496	287	306.944	331	352.437	375	397.832
288	319.153	332	365.421	376	411.543	288	307.980	332	353.469	376	398.863
289	320.206	333	366.471	377	412.590	289	309.014	333	354.501	377	399.894
290	321.259	334	367.521	378	413.636	290	310.050	334	355.534	378	400.924
291	322.313	335	368.570	379	414.683	291	311.084	335	356.567	379	401.954
292	323.366	336	369.620	380	415.729	292	312.119	336	357.599	380	402.985
293	324.418	337	370.668	381	416.776	293	313.155	337	358.632	381	404.015
294	325.472	338	371.718	382	417.822	294	314.189	338	359.664	382	405.046
295	326.524	339	372.767	383	418.868	295	315.223	339	360.697	383	406.077
296	327.577	340	373.817	384	419.915	296	316.258	340	361.729	384	407.107
297	328.630	341	374.865	385	420.961	297	317.293	341	362.761	385	408.138
298	329.683	342	375.915	386	422.007	298	318.327	342	363.793	386	409.168
299	330.735	343	376.964	387	423.054	299	319.362	343	364.826	387	410.198
300	331.788	344	378.012	388	424.100	300	320.397	344	365.858	388	411.228
301	332.840	345	379.062	389	425.146	301	321.430	345	366.890	389	412.259
302	333.892	346	380.110	390	426.191	302	322.465	346	367.922	390	413.289
303	334.945	347	381.159	391	427.237	303	323.499	347	368.954	391	414.320
304	335.996	348	382.208	392	428.284	304	324.533	348	369.986	392	415.348
305	337.048	349	383.256	393	429.329	305	325.568	349	371.018	393	416.379
306	338.100	350	384.305	394	430.375	306	326.602	350	372.049	394	417.409
307	339.152	351	385.354	395	431.420	307	327.635	351	373.082	395	418.439
308	340.204	352	386.402	396	432.466	308	328.670	352	374.113	396	419.469
309	341.256	353	387.450	397	433.512	309	329.704	353	375.145	397	420.499
310	342.307	354	388.499	398	434.557	310	330.738	354	376.177	398	421.529
311	343.358	355	389.547	399	435.603	311	331.771	355	377.209	399	422.558
312	344.410	356	390.595	400	436.648	312	332.805	356	378.240	400	423.588
313	345.461	357	391.642	401	437.693	313	333.839	357	379.271	401	424.618
314	346.512	358	392.690	402	438.739	314	334.873	358	380.303	402	425.648

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